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Some Unpublished Papers of Jean van Heijenoort

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Jean Louis van Heijenoort (1912–1986) was most widely known as the editor of *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* [1967], and it is upon this work that his reputation as an historian of logic was largely founded. He was also the editor of the logical writings of Jacques Herbrand [Herbrand 1968], a co-editor of the collected works of Kurt Gödel [Gödel 1986], and the author of a small number of original papers in history and philosophy of logic, many appearing posthumously for the first time in his *Selected Essays* [1986].

Van Heijenoort was also the author of a number of original technical (nonhistorical, nonphilosophical) papers in mathematical logic. His booklet *El desarrollo de la teoría de la cuantificación* [1976] can be seen primarily as a history of the development of quantification theory, or primarily as a critical and comparative analysis of four major approaches to first-order functional logic (the axiomatic method, the Herbrand method, the Gentzen sequent-calculus method, and the method of natural deduction). His booklet *Introduction à la sémantique des logiques non-classiques* [1979] is an extension to nonclassical (modal, intuitionistic, and three-valued) logics of results which he obtained earlier for classical quantification theory. Most of these earlier results, however, remained unpublished.

As an historian of logic, van Heijenoort was interested in the history of the

development of quantification theory, which he saw as a family of formal systems which numbered among the chief family members the axiomatic method, Herbrand quantification, the Gentzen sequent-calculus, and natural deduction. In [1976], van Heijenoort sketched the strengths and weaknesses of these four principal methods in their classical, intuitionistic, and minimal versions. He was particularly interested in the proof-theoretic strengths of these methods.

In Hilbert's axiomatic system, one is restricted by the finitism to the concept of *satisfiability*. A set of formulae is *satisfiable*, or valid only with respect to a particular finitary model, in Hilbert's system. For Herbrand quantification, we may select a universe of discourse which is either finite or infinite; and by doing so, develop fully the concept of *validity* as satisfiability for every possible model of the system. By Herbrand expansion, one may effectively obtain quantifier-free formulae from quantified formulae. Herbrand disjunction is used to eliminate existential quantifiers, and Herbrand conjunction is used to eliminate universal quantifiers. In the unpublished paper [1975b], van Heijenoort details the technical apparatus developed by Herbrand for carrying out these expansions. The quantifier-free formulae so obtained are then sententially valid, and thus are valid for every model. This work of Herbrand, therefore, helped characterize the concept of *being a proof* in terms of *validity* rather than in the model-dependent terms of *satisfiability*.

In [1968], van Heijenoort showed how the falsifiability tree method could be adapted to Herbrand expansion to test the validity of quantified formulae or sets of formulae whether or not those formulae were in prenex form. ([1970] is the published abstract of this result.) Thereafter, van Heijenoort became a staunch advocate of the falsifiability tree method, and all his original technical work in mathematical logic was designed to provide technical proof-theoretic substantiation of the superiority of this method.

It is not possible to ascertain with complete confidence the origin of the unpublished technical papers. All of them are dated between 1968 and 1975, although some appear to be revisions of earlier papers. Some of these papers are written in van Heijenoort's elaborate, stylized European script, although most are typed. Very often, the typed papers are unpolished, bearing the typographical errors of inexperienced or careless secretaries. For example, van Heijenoort's paper on Herbrand [1975b] carries on the last page of the text a handwritten note by van Heijenoort in which he says: "I apologize for the poor typing, which was done by an apparently reluctant typist." Neither what the original purpose of these papers may have been, nor whether they were ever intended for publication, is clear. I obtained my copies of the papers between the autumn of 1974 and the spring of 1976, at a time when van Heijenoort was distributing photocopies of them to his students at Brandeis University for use as supplementary readings for his courses. (The exception is [1968], which I received on 11 June 1987 from Richard C. Jeffrey, who sent me a copy of his copy.) These papers apparently did not circulate widely to van Heijenoort's colleagues, however, since, with the exception of Jeffrey, those who responded to my inquiries (for example, W. V. Quine of

Harvard, who was quite close to van Heijenoort) reported that they were unaware of the existence of any unpublished papers by van Heijenoort on the falsifiability tree method and could not find copies of such papers, published or unpublished, in their files. Since many of these papers are handwritten [1973, 1975, 1975a] and several of the typed ones are labeled as drafts [1972, 1975b], we can conclude either that these papers were not ready for publication (that is, did not meet van Heijenoort's exacting standards), or that they were never intended for publication.

The unpublished technical papers of van Heijenoort may be briefly described as follows.

[1968] *On the relation between the falsifiability tree method and the Herbrand method in quantification theory*: typescript, 12 pp. (20 November 1968):

—modification of the falsifiability tree method to accommodate Herbrand expansion whether or not quantified formulae are in prenex form.

[1972] *The falsifiability-tree method for the simple theory of types with extensionality*: typescript, 23 pp. (23 July 1972); draft:

—proof of the soundness and completeness of the falsifiability tree method for the simple theory of types with extensionality.

[1973] *Soundness and completeness of the falsifiability-tree method for sentential logic*: manuscript, 9 pp. (23 September 1973):

—proof of the soundness and completeness of the falsifiability tree method for classical propositional calculus.

[1974] *Falsifiability trees*: typescript, 25 pp. (15 March 1972; retyped with corrections, 30 September 1974):

—proof of the soundness and completeness of the falsifiability tree method for quantification theory (classical first-order functional calculus); includes a proof of König's lemma.

[1974a] (*Proof of the law of lesser universes*): untitled and undated typescript, 4 pp., originally stapled to [1974] as an additional appendix:

—the major result is a proof of Quine's law of lesser universes (Theorem 3.05 of the typescript): Let a, b be two cardinals such that $0 < a \leq b$. If a formula is a -satisfiable, then it is b -satisfiable; if it is b -valid, then it is a -valid.

[1975] *The tree method for intuitionistic sentential logic*: manuscript, 5 pp. (5 May 1975):

—proof of the soundness and completeness of the tree method for intuitionistic propositional logic. A tree for an intuitionistic formula A is inconsistent if and only if A is classically provable; every nonconsistent ramified branch of a finished tree for an intuitionistic formula A yields a Kripke model which fails to satisfy A .

[1975a] *The tree method for intuitionistic quantification theory*: manuscript, 3 pp. (9 May 1975):

—proof of the soundness and completeness of the tree method for first-order intuitionistic logic, by adding to the proof in [1975] the additional three cases of true universal quantification, true existential quantification, and false existential quantification.

[1975b] *Herbrand*: typescript, 15 pp. (18 May 1975):

—detailed account of the technical apparatus for Herbrand expansion and proof of Herbrand's *Fundamental Theorem*; proof of the soundness and completeness of Herbrand quantification.

The papers [1973–1974a] were distributed by van Heijenoort to his students as required readings for the course “Intermediate Logic” (Autumn 1974) to supplement the basic course textbook [Jeffrey 1967], which presented an introductory-level approach to logic using the tree method. [1972] was distributed in conjunction with either van Heijenoort's seminar “Theories of Quantification” (Autumn 1976), or possibly his course “Foundations of Mathematics” (Spring 1975) or his seminar on Frege (Spring 1975) in connection with a consideration of Russell's theory of types. [1975b] was distributed during the seminar on “Theories of Quantification,” and [1975, 1975a] during the “Foundations of Mathematics” course.

Apparently, van Heijenoort felt either that these technical papers were not ready for publication, or that they were too insignificant perhaps to merit publication. Nevertheless, I believe that a collection of the main body of van Heijenoort's nonphilosophical and nonhistorical papers deserves publication. At the very least, the papers would allow scholars to gauge the value of van Heijenoort's technical contributions to mathematical logic. They might also lead to a reevaluation of van Heijenoort as a working mathematical logician rather than (almost exclusively) as an historian and philosopher of logic. In addition, the publication of many of van Heijenoort's technical papers (in particular, [1973–1974a]) would benefit students using [Jeffrey 1967] as their basic text. They would find van Heijenoort's proofs of the soundness and completeness of the tree method extremely useful, since Jeffrey deals with these very informally. Publication would also benefit students using the more advanced text [Bell & Machover 1977], since van Heijenoort's results require fewer preliminaries and therefore evidence greater technical elegance than the proofs presented by Bell and Machover.

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Biography of John von Neumann

By Nicholas A. Vonneuman

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Jean van Heijenoort's contributions to proof theory and its history Anellis, Irving H., *Modern Logic*, 1992. Theory of quantification, logic and language (concerning the Selected essays of Jean van Heijenoort Padilla-Gálvez, Jesús, *Modern Logic*, 1992. Review of I. H. Anellis, Van Heijenoort -- Logic and its History in the Work and Writings of Jean van Heijenoort Băziau, Jean-Yves, *Modern Logic*, 2000. The Life of Jean van Heijenoort; Solomon Feferman, Jean van Heijenoort's Scholarly Work, 1948-1986 Enderton, H. B., *Journal of Symbolic Logic*, 1993. The Road to Modern Logic-An Interpretation Ferreiros, Jose, *Bulletin of Symbolic Logic*, 2001. Jean van Heijenoort (1912--1986) Feferman, Anita Burdman and Feferman, Solomon, *Modern Logic*, 1992. Jean van Heijenoort was best known for his editorial work in the history of mathematical logic. I survey van Heijenoort's contributions to model-theoretic proof theory, and in particular to the falsifiability tree method. This work of van Heijenoort's is not widely known, and much of it remains unpublished. View. Show abstract. Scitation is the online home of leading journals and conference proceedings from AIP Publishing and AIP Member Societies. View. Show abstract.