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Boekbespreking

R. Kane
Reflection Groups and Invariant Theory
CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, Springer-Verlag, 2001
379 p., prijs € 39,54
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Review by A. Broer

This book by the Canadian mathematician Richard Kane is an elementary exposition of the theory of finite pseudo-reflection groups and their invariant theory as developed up to the beginning of the 1980's. It does not contain new results. It can be read by a beginning graduate student, it requires almost no prerequisites and never becomes hard to read. Basically all arguments are of an easily digestible algebraic nature. There are no exercises and no historical comments, but it has a good list of references to the not too recent literature.

This is certainly not the first text-book discussing these topics. Coxeter and Weyl groups appear in any book on groups of Lie type or quantum groups or on semisimple Lie algebras or Kac-Moody algebras. Reflections and reflection groups appear in any book on Euclidean geometry, on hyperplane arrangements or on invariant theory of finite groups. Rings of covariants of Weyl groups appear as cohomology algebras of flag manifolds and are discussed in books on Schubert calculus and algebraic combinatorics. The topics are centrally important in many different and attractive fields.

The basic nature and coherence of these topics became clear in 1968 when Nicholas Bourbaki published a volume of his Éléments de mathématique containing Chapters IV, V, VI of his treatise on Lie groups and Lie algebras. Although it is embedded in a treatise on Lie theory, it is completely independent of it. In fact, Lie theory is only mentioned in the introduction and in the historical notes. It has a large number of non-trivial exercises and an appendix with explicit information on root systems. Borel calls it 'one of the most successful books by Bourbaki' (p. 379, A. Borel, *Twenty-five years with Nicolas Bourbaki, 1949-1973,* Notices of the Amer. Math. Soc. **45** (1998), 373–380).

Nevertheless, the reading (and solving of the exercises) was considered as demanding and expositions presenting part of the material on a more elementary level kept appearing. Two of the more popular examples are Grove and Benson's *Finite reflection groups*, 1971 and 1985 ('We have tried to reach a middle ground between Coxeter and Bourbaki.'), and Humphreys' *Reflection groups and Coxeter groups*, 1990 ('The present book attempts to be both an introduction to Bourbaki and an updating of the coverage (...).').

The book under review is of the same kind. It incorporates improvements published since the first appearance of Bourbaki's book, for example Springer's theory of regular elements and the notion of generalized invariant of Kac-Peterson. On the other hand some topics that did appear in Bourbaki, like Tits systems and Hecke algebras, are completely left out in the book under review. Indeed, these topics may be too far removed from invariant theory. Also, advanced topics like Springer's representations are necessarily left out, since elementary algebraic accounts are not yet available. Similarly there is no place for the more recent invariant theory of two copies of the reflection representation. This has been studied in recent years in relation with Haiman's sensational results in algebraic combinatorics based on the symmetric group.

All in all it is a carefully written book that can be recommended to a graduate student for independent study, containing a good amount of interesting material.

A reflection group is, then, any group of transformations generated by such reflections. The purpose of this book is to study such groups and their associated invariant theory, outlining the deep and elegant theory that they possess. This is a preview of subscription content, log in to check access. Preview. Unable to display preview. Download preview PDF. Copyright information.Â Kane R., Borwein J., Borwein P. (2001) Introduction: Reflection groups and invariant theory. In: Borwein J., Borwein P. (eds) Reflection Groups and Invariant Theory. CMS Books in Mathematics (Ouvrages de mathÃ©matiques de la SMC). Springer, New York, NY. DOI https://doi.org/10.1007/978-1-4757-3542-0_1. Publisher Name Springer, New York, NY. Print ISBN 978-1-4419-3194-8. "This book by the Canadian mathematician Richard Kane is an elementary exposition of the theory of finite pseudo-reflection groups â€¦ . It can be read by a beginning graduate student, it requires almost no prerequisites and never becomes hard to read. â€¦ it has a good list of references to the not too recent literature. â€¦ All in all it is a carefully written book that can be recommended to a graduate student for independent study, containing a good amount of interesting material." (A. Broer, Nieuw Archief voor Wiskunde, Vol. 6 (2), 2005). "The main theme of this book is a study of CMS Books in Mathematics, Ouvrages de MathÂ´ematiques de la SMC, 5. Springer-Verlag, New York, 2001. [362] S. Kantor, Theorie der endlichen Gruppen von eindeutigen Transformationen in der Ebene , Berlin. Mayer & Mller. 1895. [363] S. Kantor, Ueber Collineationsgruppen an Kummerâ€™schen Flachen , Amer. J. Math. 19 (1897), 86â€“91.Â [365] P. Katsylo, On the birational geometry of the space of ternary quartics . Lie groups, their discrete subgroups, and invariant theory, 95â€“103, Adv. Soviet Math., 8 , Amer. Math. Soc., Providence, RI, 1992. [366] P. Katsylo, On the unramiï¬ed 2-covers of the curves of genus 3 , in Algebraic Geometry and its Applications (Yaroslavl, 1992) , Aspects of Mathematics, vol. E25, Vieweg, 1994, pp. 61â€“65.