To our parents
Contents

PREFACE XIII

ABOUT THE AUTHOR XVI

1 FUNDAMENTAL CONCEPTS 1

1.1 Introduction 1
1.2 Historical Background 1
1.3 Outline of Presentation 2
1.4 Stresses and Equilibrium 2
1.5 Boundary Conditions 4
1.6 Strain–Displacement Relations 5
1.7 Stress–Strain Relations 6
   Special Cases 7
1.8 Temperature Effects 8
1.9 Potential Energy and Equilibrium: The Rayleigh–Ritz Method 9
   Potential Energy Π 9
   Rayleigh–Ritz Method 12
1.10 Galerkin’s Method 14
1.11 Saint Venant’s Principle 18
1.12 Von Mises Stress 19
1.13 Principle of Superposition 19
1.14 Computer Programs 20
1.15 Conclusion 20
   Historical References 20
   Problems 21

2 MATRIX ALGEBRA AND GAUSSIAN ELIMINATION 28

2.1 Matrix Algebra 28
   Row and Column Vectors 29
vi Contents

Addition and Subtraction, 29
Multiplication by a Scalar, 29
Matrix Multiplication, 29
Transposition, 30
Differentiation and Integration, 30
Square Matrix, 31
Diagonal Matrix, 31
Identity Matrix, 31
Symmetric Matrix, 32
Upper Triangular Matrix, 32
Determinant of a Matrix, 32
Matrix Inversion, 32
Eigenvalues and Eigenvectors, 33
Positive Definite Matrix, 35
Cholesky Decomposition, 35

2.2 Gaussian Elimination 35
   General Algorithm for Gaussian Elimination, 37
   Symmetric Matrix, 40
   Symmetric Banded Matrices, 40
   Solution with Multiple Right Sides, 40
   Gaussian Elimination with Column Reduction, 42
   Skyline Solution, 44
   Frontal Solution, 45

2.3 Conjugate Gradient Method for Equation Solving 45
   Conjugate Gradient Algorithm, 46
   Input Data/Output 46

Problems 47
   Program Listings, 49

3 ONE-DIMENSIONAL PROBLEMS 51

3.1 Introduction 51
3.2 Finite Element Modeling 52
   Element Division, 52
   Numbering Scheme, 53
3.3 Shape Functions and Local Coordinates 55
3.4 The Potential-Energy Approach 59
   Element Stiffness Matrix, 60
   Force Terms, 62
3.5 The Galerkin Approach 64
   Element Stiffness, 64
   Force Terms, 65
3.6 Assembly of the Global Stiffness Matrix and Load Vector 66
3.7 Properties of $K$ 69
3.8 The Finite Element Equations: Treatment of Boundary Conditions 70
Contents

3.9 Quadratic Shape Functions  85
3.10 Temperature Effects  92
3.11 Problem Modeling and Boundary Conditions  96
  Problem in Equilibrium, 96
  Symmetry, 97
  Two Elements with Same End Displacements, 97
  Problem with a Closing Gap, 98
  Input Data/Output, 98

Problems  99
  Program Listing, 111

4 TRUSSES  117
  Introduction  117
  Plane Trusses  118
    Local and Global Coordinate Systems, 118
    Formulas for Calculating $\ell$ and $m$, 119
    Element Stiffness Matrix, 120
    Stress Calculations, 121
    Temperature Effects, 126
  Three-Dimensional Trusses  129
  Assembly of Global Stiffness Matrix for the Banded and Skyline Solutions  131
    Assembly for Banded Solution, 131
    Skyline Assembly, 132
  Problem Modeling and Boundary Conditions  134
    Inclined Support in Two Dimensions, 134
    Inclined Support in Three Dimensions—Line Constraint, 134
    Inclined Support in Three Dimensions—Plane Constraint, 135
    Symmetry and Antisymmetry, 136
    Input Data/Output, 138

Problems  139
  Program Listing, 147

5 BEAMS AND FRAMES  150
  Introduction  150
    Potential-Energy Approach, 151
    Galerkin Approach, 152
  Finite Element Formulation  153
    Element Stiffness—Direct Approach, 157
  Load Vector  158
5.4 Boundary Considerations 159
5.5 Shear Force and Bending Moment 160
5.6 Beams on Elastic Supports 162
5.7 Plane Frames 163
5.8 Three-Dimensional Frames 169
5.9 Problem Modeling and Boundary Conditions 173
5.10 Some Comments 174
    Input Data/Output, 174
    Problems 176
    Program Listings, 183

6 TWO-DIMENSIONAL PROBLEMS USING CONSTANT STRAIN TRIANGLES 188

6.1 Introduction 188
6.2 Finite Element Modeling 189
6.3 Constant Strain Triangle (CST) 191
    Isoparametric Representation, 192
    Potential-Energy Approach, 198
    Element Stiffness, 198
    Force Terms, 199
    Integration Formula on a Triangle, 206
    Galerkin Approach, 206
    Stress Calculations, 208
    Temperature Effects, 210

6.4 Problem Modeling and Boundary Conditions 212
    Some General Comments on Dividing into Elements, 215

6.5 Patch Test and Convergence 215
    Patch Test, 215

6.6 Orthotropic Materials 216
    Temperature Effects, 220
    Input Data/Output, 222

    Problems 225
    Program Listing, 238

7 AXISYMMETRIC SOLIDS SUBJECTED TO AXISYMMETRIC LOADING 242

7.1 Introduction 242
7.2 Axisymmetric Formulation 243
7.3 Finite Element Modeling: Triangular Element 245
    Potential-Energy Approach, 248
    Body Force Term, 249
    Rotating Flywheel, 249
    Surface Traction, 250
    Galerkin Approach, 252
Contents ix

Stress Calculations, 255
Temperature Effects, 256

7.4 Problem Modeling and Boundary Conditions 256
   Cylinder Subjected to Internal Pressure, 256
   Infinite Cylinder, 257
   Press Fit on a Rigid Shaft, 257
   Press Fit on an Elastic Shaft, 258
   Belleville Spring, 259
   Thermal Stress Problem, 260
   Input Data/Output, 262

Problems 263
   Program Listing, 271

8 TWO-DIMENSIONAL ISOPARAMETRIC ELEMENTS
   AND NUMERICAL INTEGRATION 273

8.1 Introduction 273
8.2 The Four-Node Quadrilateral 273
   Shape Functions, 273
   Element Stiffness Matrix, 276
   Element Force Vectors, 279
8.3 Numerical Integration 279
   Two-Dimensional Integrals, 283
   Stiffness Integration, 283
   Stress Calculations, 284
8.4 Higher Order Elements 286
   Nine-Node Quadrilateral, 287
   Eight-Node Quadrilateral, 289
   Six-Node Triangle, 290
   Integration on a Triangle—Symmetric Points, 291
   Integration on a Triangle—Degenerate Quadrilateral, 292
8.5 Four-Node Quadrilateral for Axisymmetric Problems 294
8.6 Conjugate Gradient Implementation of the Quadrilateral Element 295
8.7 Concluding Remarks and Convergence 295
8.8 References for Convergence 297
   Input Data/Output, 298

Problems 300
   Program Listings, 308

9 THREE-DIMENSIONAL PROBLEMS IN STRESS ANALYSIS 312

9.1 Introduction 312
9.2 Finite Element Formulation 313
   Element Stiffness, 316
   Force Terms, 317

Contents

9.3 Stress Calculations 317
9.4 Mesh Preparation 318
9.5 Hexahedral Elements and Higher Order Elements 322
9.6 Problem Modeling 324
9.7 Frontal Method for Finite Element Matrices 326
   Connectivity and Prefront Routine, 327
   Element Assembly and Consideration of Specified dof, 328
   Elimination of Completed dof, 328
   Backsubstitution, 329
   Consideration of Multipoint Constraints, 329
   Input Data/Output, 330

Problems 332
   Program Listings, 336

10 SCALAR FIELD PROBLEMS 345

10.1 Introduction 345
10.2 Steady State Heat Transfer 346
   One-Dimensional Heat Conduction, 347
   One-Dimensional Heat Transfer in Thin Fins, 355
   Two-Dimensional Steady-State Heat Conduction, 359
   Two-Dimensional Fins, 369
   Preprocessing for Program Heat2D, 370
10.3 Torsion 370
   Triangular Element, 372
   Galerkin Approach, 373
10.4 Potential Flow, Seepage, Electric and Magnetic Fields, and Fluid Flow in Ducts 376
   Potential Flow, 376
   Seepage, 378
   Electrical and Magnetic Field Problems, 379
   Fluid Flow in Ducts, 381
   Acoustics, 383
   Boundary Conditions, 384
   One-Dimensional Acoustics, 384
   One-Dimensional Axial Vibrations, 386
   Two-Dimensional Acoustics, 388
10.5 Conclusion 389
   Input Data/Output, 389

Problems 391
   Program Listings, 402

11 DYNAMIC CONSIDERATIONS 408

11.1 Introduction 408
11.2 Formulation 408
   Solid Body with Distributed Mass, 409

11.3 Element Mass Matrices 411
11.4 Evaluation of Eigenvalues and Eigenvectors 416
   Properties of Eigenvectors, 417
   Eigenvalue–Eigenvector Evaluation, 417
   Inverse Iteration Method, 420
   Generalized Jacobi Method, 423
   Tridiagonalization and Implicit Shift Approach, 427
   Bringing Generalized Problem to Standard Form, 427
   Tridiagonalization, 428
   Implicit Symmetric QR Step with Wilkinson Shift
   for Diagonalization, 431
11.5 Interfacing with Previous Finite Element Programs and a Program
   for Determining Critical Speeds of Shafts 432
11.6 Guyan Reduction 433
11.7 Rigid Body Modes 436
11.8 Conclusion 438
   Input Data/Output, 438
   Problems 440
   Program Listings, 446

12  PREPROCESSING AND POSTPROCESSING 453

12.1 Introduction 453
12.2 Mesh Generation 453
   Region and Block Representation, 453
   Block Corner Nodes, Sides, and Subdivisions, 454
12.3 Postprocessing 461
   Deformed Configuration and Mode Shape, 461
   Contour Plotting, 462
   Nodal Values from Known Constant Element Values
   for a Triangle, 463
   Least-Squares Fit for a Four-Noded Quadrilateral, 465
12.4 Conclusion 466
   Input Data/Output, 467
   Problems 468
   Program Listings, 470

APPENDIX  Proof of \( dA = \det J \, d\xi \, d\eta \) 483

BIBLIOGRAPHY 486

ANSWERS TO SELECTED PROBLEMS 490

INDEX 492

Preface

The first edition of this book appeared over 20 years ago and the second and third editions followed subsequently. Translations of the book appeared in Spanish, Korean, Greek, and Chinese languages. We received positive feedback from professors who taught from the book and from students and practicing engineers who used the book. We also benefited from the feedback received from the students in our courses for the past 30 years. We have incorporated several suggestions in this edition. The underlying philosophy of the book is to provide a clear presentation of theory, aspects of problem modeling and implementation into computer programs. The pedagogy of earlier editions has been retained and enhanced in this edition.

WHAT'S NEW IN THIS EDITION

• Introduction of the superposition principle.
• Treatment of symmetry and antisymmetry.
• Additional examples and exercise problems.
• The patch test.
• Beams and Frames chapter moved to follow the Truss chapter.
• Revised Excel VB programs.
• Programs in JAVASCRIPT to run on web browsers such as IE, Firefox, Google Chrome, or Safari.
• Executable graphics programs together with the source codes.
• Additional examples and exercise problems.

New material has been introduced in several chapters. Worked examples and exercise problems have been added to supplement the learning process. Exercise problems stress both fundamental understanding and practical considerations. Problem modeling aspects have been added from early chapters. Principle of superposition is introduced in Chapter 1. Symmetry and antisymmetry considerations in two-dimensional problems are clearly presented. There are additional examples and exercise problems. The patch test is discussed and related problems have been added. The included
programs have a common structure, which should enable the users to follow the
development easily. Programs in JAVASCRIPT have been added. This will enable the
users to solve finite element analysis problems using web browsers such as IE, Firefox,
Safari, or Google Chrome. Excel VB programs have been revised. All programs have been
thoroughly checked. The downloadable program set includes executable versions for
programs involving graphics. The programs have been provided in Visual Basic, Microsoft
Excel/Visual Basic, MATLAB, and JAVASCRIPT, together with those provided earlier in
QBASIC, FORTRAN, and C. The Solutions Manual has been updated.

Chapter 1 gives a brief historical background and develops the fundamental
concepts. Equations of equilibrium, stress–strain relations, strain–displacement
relations, and the principles of potential energy are reviewed. The concept of Galerkin's
method is introduced.

Properties of matrices and determinants are reviewed in Chapter 2. The Gaussian
elimination method is presented, and its relationship to the solution of symmetric
banded matrix equations and the skyline solution is discussed. Cholesky decomposition
and conjugate gradient method are discussed.

Chapter 3 develops the key concepts of finite element formulation by considering
one-dimensional problems. The steps include development of shape functions,
derivation of element stiffness, formation of global stiffness, treatment of boundary
conditions, solution of equations, and stress calculations. Both the potential energy
approach and Galerkin's formulations are presented. Consideration of temperature
effects is included.

Finite element formulation for plane and three-dimensional trusses is developed
in Chapter 4. The assembly of global stiffness in banded and skyline forms is explained.
Computer programs for both banded and skyline solutions are given.

Beams and application of Hermite shape functions are presented in Chapter 5.
The chapter covers two-dimensional and three-dimensional frames.

Chapter 6 introduces the finite element formulation for two-dimensional plane
stress and plane strain problems using constant strain triangle (CST) elements. Problem
modeling and treatment of boundary conditions are presented in detail. Formulation
for orthotropic materials is provided.

Chapter 7 treats the modeling aspects of axisymmetric solids subjected to
axisymmetric loading. Formulation using triangular elements is presented. Several
real-world problems are included in this chapter.

Chapter 8 introduces the concepts of isoparametric quadrilateral and higher-
order elements and numerical integration using Gaussian quadrature. Formulation for
axisymmetric quadrilateral element and implementation of conjugate gradient method
for quadrilateral element are given.

Chapter 9 presents three-dimensional stress analysis. Tetrahedral and hexahedral
elements are presented. The frontal method and its implementation aspects are
discussed.

Scalar field problems are treated in detail in Chapter 10. While Galerkin as well
as energy approaches have been used in every chapter with equal importance, only
Galerkin's approach is used in this chapter. This approach directly applies to the given
differential equation without the need of identifying an equivalent functional to
minimize. Galerkin’s formulation for steady-state heat transfer, torsion, potential flow, seepage flow, electric and magnetic fields, fluid flow in ducts, and acoustics are presented.

Chapter 11 introduces dynamic considerations. Element mass matrices are given. Techniques for evaluation of eigenvalues (natural frequencies) and eigenvectors (mode shapes) of the generalized eigenvalue problem are discussed. Methods of inverse iteration, Jacobi, tridiagonalization, and implicit shift approaches are presented.

Preprocessing and postprocessing concepts are developed in Chapter 12. Theory and implementation aspects of two-dimensional mesh generation, least-squares approach to obtain nodal stresses from element values for triangles and quadrilaterals, and contour plotting are presented.

At the undergraduate level some topics may be dropped or delivered in a different order without breaking the continuity of presentation. We encourage the introduction of the Chapter 12 programs at the end of Chapter 6. This helps the students to prepare the data in an efficient manner.

We thank Professor Hongbing Fang, Mechanical Engineering and Engineering Science, UNC Charlotte; Professor Kishore Pochiraju, Department of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey; Professor Subramaniam Rajan, Ira A. Fulton School of Engineering, Arizona State University; Professor Chris H. Reidel, A. Leon Linton Department of Mechanical Engineering, Lawrence Technological University, Michigan; and Professor Nicholas J. Zabaras, Sibley School of Mechanical and Aerospace Engineering, Cornell University, who reviewed our third edition and gave many constructive suggestions that helped us improve the book.

Complete self-contained computer programs with source codes in Visual Basic, Excel-based Visual Basic, MATLAB, FORTRAN, JAVASCRIPT and C to accompany the text are available at www.pearsonhighered.com/chandrupatla.

Tirupathi Chandrupatla expresses his gratitude to J. Tinsley Oden, whose teaching and encouragement have been a source of inspiration to him throughout his career. He expresses his thanks to many students at Rowan University and Kettering University who took his courses. He expresses his thanks to his colleague Paris von Lockette who gave valuable feedback after teaching the course using the second and third editions.

Ashok D. Belegundu thanks his students at Penn State for their feedback on the course material and programs.

We thank Marcia Horton who provided us guidance with the present and previous editions of the book. We thank our editor Norrin Dias, Tacy Quinn, Debbie Yarnell, and Clare Romeo all from Prentice Hall who turned this project into a pleasant chore. We thank Maheswari PonSaravanan, project manager and her team at TexTech International, Chennai, India for efficient handling of copyediting and page proofs.

About the Authors

Tirupathi R. Chandrupatla is Professor and Founding Chair of Mechanical Engineering at Rowan University, Glassboro, New Jersey. He received the B.S. degree from National Institute of Technology (formerly REC), Warangal. He received the M.S. degree in design and manufacturing from the Indian Institute of Technology (IIT), Bombay. He started his career as a design engineer with Hindustan Machine Tools, Bangalore. He then taught in the Department of Mechanical Engineering at IIT, Bombay. He pursued his graduate studies in the Department of Aerospace Engineering and Engineering Mechanics at the University of Texas at Austin and received his Ph.D. in 1977. He subsequently taught at the University of Kentucky. Prior to joining Rowan, he was a Professor of Mechanical Engineering and Manufacturing Systems Engineering at Kettering University (formerly GMI Engineering and Management Institute/General Motors Institute), where he taught for 16 years.

Dr. Chandrupatla has broad research interests, which include finite element analysis, design, optimization, quality and reliability, and manufacturing engineering. He has published widely in these areas and serves as a consultant to industry. Dr. Chandrupatla is a registered Professional Engineer and also a Certified Manufacturing Engineer. He is a member of ASEE, ASME, SAE, and SME.

Ashok D. Belegundu has been a Professor of Mechanical Engineering at The Pennsylvania State University, University Park, since 1986. Prior to this, he taught at GMI, now Kettering University, in Michigan. He received his B.Tech. degree from IIT, Madras and his Ph.D. from the University of Iowa. He has been a principal investigator on research projects involving finite element based optimization for several agencies, including the National Science Foundation, Army Research Office, NASA, SERC (UK), MacNeal-Schwendler Corporation, Gentex Corporation, and Ingersoll-Rand. A detailed list of his publications and projects can be found at http://www.mne.psu.edu/Directories/Faculty/Belegundu-A.html. He has given short courses on finite elements to the Forging Industry Association, Hazleton Pumps, Infosys (India).
Introduction to Finite Elements in Engineering is ideal for senior undergraduate and first-year graduate students and also as a learning resource to practicing engineers. This book provides an integrated approach to finite element methodologies. The development of finite element theory is combined with examples and exercises involving engineering applications. The steps used in the development of the theory are implemented in complete, self-contained computer programs. While the strategy and philosophy are introduced. Axisymmetric Formulation. Finite Element Modeling: Triangular Element. Problem Modeling and Boundary Conditions. 7. Two-Dimensional Isoparametric Elements and Numerical Integration. Prior to joining Rowan, he was a Professor of Mechanical Engineering and Manufacturing Systems Engineering at GMI Engineering & Management Institute (formerly General Motors Institute), where he taught for 16 years. Dr. Chandrupatla has broad research interests, which include finite element analysis, design, optimization, and manufacturing engineering. He has published widely in these areas and serves as a consultant to industry. Dr. Chandrupatla is a registered Professional Engineer and also a Certified Manufacturing Engineer.