

Book reviews

1. Elements of Stochastic Modelling

by K. Borovkov, World Scientific, New Jersey – London – Singapore – HongKong, 2003, XIII + 342 pp., ISBN 981 - 238 - 300 - X, 981 - 238 - 301 - 8 (pbk)

Contents : Preface; 1. Introduction; 2. Basics of probability theory; 3. Markov chains; 4. Markov decision processes; 5. The exponential distribution and Poisson processes; 6. Jump Markov processes; 7. Elements of queueing theory; 8. Elements of renewal theory; 9. Elements of time series; 10. Elements of simulation; Answers to problems; Greek alphabet; Notations; Abbreviations; Index.

The title is somewhat misleading since in fact this is an introductory course on stochastic processes: emphasis is here on stochastic MODELS (as an equivalent term for stochastic processes) and not on the process (or art !) of stochastic MODELLING.

The presentation of the material is very clear throughout. Proofs are often just sketched while the reader is referred to more advanced texts for both complete proofs and further developments of the topics considered.

The value of the book is greatly enhanced by the problems for solving (with answers) at the end of Chapters 2 through 9, and also by the many comments and historical notes included.

We think that this book is a valuable addition to the textbook literature on stochastic processes.

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2. Analytic Methods in the Theory of Differential and Pseudo-Differential Equations of Parabolic Type

by Samuil D. Eidelman, Stepan D. Ivasyshen, Anatoly N. Kochubei. Operator Theory: Advances and Applications, Vol. 152, Birkhäuser Verlag, Basel-Boston-Berlin, ix+386 pp., 2004, ISBN 3-7643-7115-3.

This book gives an overview of analytic methods of constructing, investigating, and using fundamental solutions of Cauchy problem for a large class of linear parabolic equations. A growing interest in this field is on the one hand due to the depth and complexity of mathematical problems emerging here and on the other hand to the importance in specific applied problems of natural science, technology, and economics. There are four main classes of linear parabolic equations discussed in this book:

- $\vec{2}b$ -parabolic partial differential equations (parabolic equations of a quasi-homogeneous structure), in which every spatial variable may have its own weight with respect to the time variable.
- degenerate partial differential equations of Kolmogorov's structure, which generalize classical Kolmogorov equations of diffusion with inertia.

- pseudo-differential equations with non-smooth quasi-homogeneous symbols.
- fractional diffusion equations.

The study of these four classes of equations follows the same general line: first, a fundamental solution of the Cauchy problem is constructed and studied, then it is used in order to investigate existence, uniqueness, and correctness.

The four classes considered in this book are different generalizations of the classical equations in the Petrovsky sense [1] that reveal us an interesting comparative analysis of the methods and results. This book is an excellent source of mathematical models for various diffusion phenomena being recommended to researchers in Applied Mathematics and to physicists interested in diffusion processes.

References

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3. Reconstructive Integral Geometry

by Victor Palamodov, Monographs in Mathematics, Vol. 98, Birkhäuser Verlag, Basel-Boston-Berlin, xii+164pp., 2004, ISBN 3-7643-7129-3.

This book aims to present, in an unitary way both well-known and new results in the reconstructive integral geometry by using explicit analytic methods. This theory was born one hundred years ago when H. Minkowski raised the problem of finding an even function f defined on the sphere S^2 knowing its integrals on big circles. Nowadays the growing interest in this field is motivated by the large number of modalities of image reconstruction such as X-ray, gamma and positron radiography, ultrasound, seismic tomography and others. The book contains 8 chapters and is an extended version of a lecture course taught by the author at Tel Aviv University. The first chapter presents a short and concise basic course in distribution theory and harmonic analysis. Chapters 2-8 contain interesting topics: Radon and Funk Transform, Reconstruction from Line Integrals, Flat Integral Transform, Incomplete Data Problems, Spherical Transform and Inversion, Algebraic Integral Transform.

The results and methods in this book are of interest to graduate students and researchers working in image reconstruction, inverse problem, and functional analysis.

The book makes the reader more familiar with the concepts used in this theory by giving him a rigorous support in this field.

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4. Advances in Pseudo-Differential Operators

by Ryuichi Ashino, Paolo Boggiatto, M.W. Wong (Editors). *Operator Theory: Advances and Applications*, Vol. 155, Birkhäuser Verlag, Basel-Boston-Berlin, vii+233pp., 2004, ISBN 3-7643-7140-4.

This book is a collection of thirteen articles presented in the special session on pseudo-differential operators given at the Fourth Congress of the International Society for Analysis, its Applications and Computation (ISAAC) held at York University in Toronto, August 11-16, 2003. This volume aims to present pseudo-differential operators in different contexts such as applications in partial differential equations, quantization in terms of Weyl transforms and Wick operators, filters and localization operators in signal analysis, numerical experiments in wavelet transform and orthonormal wavelet bases. Chapter 1 presents a short survey on the methods of Micro Analysis and gives a new formula for the composition of Wick operators. Chapter 2 develops the calculus of boundary value problems on a manifold with edges in parameter-dependent form. Chapters 3-5 deal with pseudo-differential operators in the context of differential equations. In Chapters 6, 7 pseudo-differential operators are presented in connection with Weyl transform encountered in quantum mechanics. In Chapters 8, 9 a study of Localization operators is given and Chapter 10 is concerned with convolutions and embeddings for weighted modular spaces. In Chapters 11-13, pseudo-differential operators are presented within the framework of numerical experiments.

This book is intended for graduated students and researchers in pure and applied mathematics as well as for physicists and engineers.

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5. Nonlinear Evolution Equations and Related Topics

by Wolfgang Arendt, Haïm Brezis, Michel Pierre (Editors), Birkhäuser Verlag, Basel-Boston-Berlin, xii+807pp., 2004, ISBN 3-7643-7107-2.

This volume is dedicated to *Philippe Bénéilan* (1940-2001) and contains 36 high quality research papers related to his work. Most of these contributions were published in *Journal of Evolution Equations*, Vol. 3, 2003. A part of them were also presented at the conference "*Journées d'Analyse Non-Linéaire*" which took place in French Jura, October 2000, on the occasion of his 60th birthday.

Among the authors who brought their contribution to this volume, we find important names in the domain of Nonlinear Analysis: H. Brezis, M. Crandall, J. Carillo, J.I. Díaz, J.A. Goldstein, M. Marcus, M. Pierre, J.L. Vázquez, L. Véron.

The main topics approached in the book are Hamilton-Jacobi equations, the porous medium equation, reaction diffusion systems, integro-differential equations and viscoelasticity, minimization problems for eigenvalues of the Laplacian, maximal L^p regularity for elliptic and parabolic equations, existence and uniqueness for large solutions of general nonlinear elliptic equations, Ornstein-Uhlenbeck operator. In this volume a particular attention is paid to the legendary work of Bénéilan-Brezis *Nonlinear problems related to the Thomas-Fermi equation* published for the first time.

The articles are carefully elaborated, skilfully combining the rigorous mathematical style with a large domain of applications.

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6. Analysis I

by Herbert Amann and Joachim Escher, Birkhäuser Verlag, Basel-Boston-Berlin, xv+426pp., 2004, ISBN 3-7643-7153-6.

This book is the first part of a three volume series which is intended to offer a comprehensive introduction to elementary Mathematical Analysis. The book is self-contained and the authors have taken a lot of care to make their exposition as clear as possible.

The book is divided into five distinct parts: Foundations, Convergence, Continuous functions, Differentiation in One Variable, and Sequences of Functions. The first part is devoted to some basic notions, such as: sets, order relations, natural and real numbers, complex numbers and linear spaces. The 12 sections of the first chapter of the book are the following: Fundamentals of Logic; Sets; Functions; Relations and Operations; The Natural Numbers; Countability; Groups and Homomorphisms; Rings, Fields and Polynomials; The Rational Numbers; The Real Numbers; The Complex Numbers and Vector Spaces, Affine Spaces and Algebras.

Part 2 is divided into 9 sections and deals with the main properties related to the notions of convergence of sequences. The sections of the second chapter of the book are the following: Convergence of Sequences; Real and Complex Sequences; Normed Vector Spaces; Monotone Sequences; Infinite Limits; Completeness; Series; Absolute Convergence and Power Series.

The third chapter of the monograph contains 6 sections and develops basic properties on topology and convergence of functions. These sections are related to: Continuity; The Fundamentals of Topology; Compactness; Connectivity; Functions on \mathbb{R} ; The Exponential and Related Functions.

Part 4 is composed of 4 sections and is devoted to the main properties of differential functions on the real axis: Differentiability; The Mean Value Theorem and its Applications; Taylor's Theorem and Iterative Procedures.

The last part of the book is intended to be an introduction to the study of analytic functions. The 4 sections of this chapter are the following: Uniform Convergence; Continuity and Differentiability for Sequences of Functions; Analytic Functions and Polynomial Approximation.

This monograph is a good overview of several concepts in modern Mathematical Analysis and it will be of interest to a broad category of specialists in the field. It can also be read by students, since only the basics of Mathematical Analysis and Linear Algebra are required. This book is a pleasure to read. It will be an excellent source,

allowing the reader to build a proper intuition and to understand the basic facts of the theory. The level of mathematical knowledge required is very modest, so it should be useful for students at undergraduate and graduate level, while researchers may find the new concepts very motivating.

References

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7. Mathematical Olympiad Treasures

by Titu Andreescu and Bogdan Enescu, Birkhäuser Verlag, Basel - Boston - Berlin, 234 pp., 2004, ISBN 0-8176-4305-2.

The book is mainly addressed to high school students in view of preparation for possible participation in mathematical competitions. This work contains carefully selected problems in Algebra, Geometry and Trigonometry, and Number Theory and Combinatorics. Most of these problems have been used in the training and testing of the USA International Mathematical Olympiad team. The book is mainly intended to offer the principal skills and techniques for solving problems in elementary Mathematics. The topics covered by the present book include: combinatorial arguments and identities, recursive relations, sums and products, number theory, polynomials, theory of equations, complex numbers in geometry, combinatorial and advanced geometry, functional equations and classical inequalities, convex functions, Cauchy-Schwartz type inequalities etc.

This book contains a wealth of nice problems and ideas for further reflection and can be recommended to all interested readers. The text covers other aspects of the subject, including several comments and remarks. This makes the book ideal for a general mathematical audience, students, and researchers from various fields of Mathematics. Knowing the good results of Romanian high school students to international mathematical competitions, we warmly recommend to them this book, as a very helpful instrument in their training.

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8. Mathematical Olympiad Challenges

by Titu Andreescu and Răzvan Gelca, Birkhäuser Verlag, Basel - Boston - Berlin, xv+260 pp., 2003, ISBN 3-7643-4155-6.

The book is a rich collection of difficult problems in elementary Mathematics. This book is intended to build a bridge between ordinary high school or undergraduate exercises and more difficult and abstract concepts or problems. In this volume it is presented an unusual collection of creative problems in Geometry, Trigonometry, Algebra, Analysis, Number Theory and Combinatorics. The authors intend to develop

some basic principles and solving techniques. They also offer a systematic illustration of how to organize the natural transition from problem solving activity towards exploring, investigating and discovering new results and properties.

Each section is self-contained, independent of the others, and focuses on one main idea. All sections start with a short discussion on the basic facts needed in the sequel, followed by some representative examples. Next, several carefully chosen problems are listed, to be solved by the readers. Complete solutions to all problems are given in the second part of the book.

The volume contains a comprehensive collection of challenging problems, the authors' goal being two-fold: first, to encourage the readers to move away from routine exercises and memorized algorithms toward creative solutions and non-standard problem solving techniques; and, second, to help our readers to develop a host of new mathematical tools and strategies that will be useful beyond the classroom and in a number of applied disciplines.

The key features of this volume are the following:

- contains a collection of challenging problems in elementary Mathematics;
- is self-contained and assumes only a basic knowledge but opens the path to competitive research in the field;
- uses competition-like problems as a platform for training typical inventive skills;
- develops basic valuable techniques for solving problems in mathematical analysis on the real axis;
- includes interesting and valuable historical account of ideas and methods in Algebra, Analysis, Geometry and Number Theory.

Last, but not least, we recall the quote chosen by the authors: “Mathematics, mathematics, mathematics, that much mathematics? No, even more”. These unforgettable words are due to the celebrated Romanian Mathematician Grigore C. Moisil.

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9. Introduction to Modern Analysis

by Shmuel Kantorovitz, Oxford University Press, Series Oxford Graduate Texts in Mathematics No. 8, Oxford, xii+434 pp., 2003, ISBN 0-19-852656-3.

This book is developed on the lectures given by the author since 1964 at Yale University, the University of Illinois at Chicago, and Bar Ilan University. This monograph contains old and new basic results from a significant part of the modern Mathematical Analysis. The author is a well-known expert in the field and his approach in this book is very elegant. All results are presented in an elementary way. Only a basic knowledge of basic functional analysis, topology and analysis is assumed. The book is well written and contains a wealth of material. The author makes a concerted effort to simplify proofs taken from many sources. That is why the researchers will readily find the information they seek, while students can develop their skills by filling in details of proofs, as well as by using the problem sets that end each chapter. The book is lovingly and carefully written. It is a delightful read.

The ten chapters of this monograph are the following: Measures, Construction of measures, Measure and topology, Continuous linear functionals, Duality, Bounded operators, Banach algebras, Hilbert spaces, Integral representation, and Unbounded operators. The book is essentially self-contained, but some relatively more difficult

topics are also included in the monograph. We mention the sections devoted to the Riesz-Markov representation theorem, Haar measure, Marcinkiewicz's interpolation theorem, the Gelfand-Naimark-Segal representation theorem, the Von Neumann double commutant theorem and the spectral representation theorem for normal operators. The monograph also contains two additional chapters which are devoted to some elementary notions and properties in Probability Theory and Distributions.

The monograph is primarily aimed at graduate and post-graduate students of Mathematics who are interested in a conceptually and mathematically clear and precise understanding of basic mathematical problems. This book is also strongly recommended to researchers in Applied Mathematics who study various linear and nonlinear phenomena but it may be also useful and comprehensive for a broader community of mathematicians, physicists, and engineers. We deeply consider that the present book is one of the most important works in its field that have been written in the last two decades. To conclude, this book provides the reader with an in-depth introduction to a rich and rapidly developing research area that has already produced remarkable results.

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10. The Novikov Conjecture, Geometry and Algebra

by Matthias Kreck, Wolfgang Lück, Oberwolfach Seminars, Vol. 33, Birkhäuser Verlag, Basel - Boston - Berlin, xvi+271 pp., 2005, ISBN 3 - 7643 - 7141 - 2.

The book under review contains lecture notes concerning a guided tour to the Novikov conjecture and related conjectures due to Baum-Connes, Borel and Farrell-Jones. It begins with basics about higher signatures, Whitehead torsion and the s -cobordism theorem. Then an introduction to surgery theory and a version of the assembly map is presented. Using the solution of the Novikov conjecture for special groups some applications to the classification of low dimensional manifolds are given. Finally, the most recent developments concerning these conjectures are surveyed, including a detailed status report.

The prerequisites consist of a solid knowledge of the topics about manifolds, vector bundles, (co-) homology and characteristic classes.

The volume contains 26 chapters including also a collection of exercises and hints for their solutions. It ends with the schedules and the list of participants to these lecture notes.

In the whole book, the theorems are exactly stated and the proofs are carefully written. In chapter after chapter, the authors take a set of theorems and, by appropriate concatenation plus some well chosen words of explanation, they create a Theory. Through the publication of this workshop in the series Oberwolfach Seminars, the Publisher, Birkhäuser Verlag, has done a fine and useful job, since the material becomes available to an even larger audience.

References

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11. Recent Advances in Operator Theory, Operator Algebras, and their Applications

XIXth International Conference on Operator Theory, Timișoara (Romania), 2002 by D. Gașpar, I. Gohberg, D. Timotin, F.H. Vasilescu, L. Zsidó, Editors, *Operator Theory: Advances and Applications*, Vol. 153, Birkhäuser Verlag, Basel - Boston - Berlin, xiv+344 pp., 2005, ISBN 3 - 7643 - 7127 - 7.

This volume consists of a careful selection of peer-reviewed articles originating from the 19th International Conference on Operator Theory, held in Timișoara in Summer 2002.

The articles contain recent developments concerning a broad range of topics from operator theory, operator algebras, and their applications, particularly to differential analysis, complex functions, ergodic theory, mathematical physics, matrix analysis, and systems theory.

The book contains a forward, the program and the list of participants at the Conference above, and the following research articles:

- (1) H. Akça, V. Covachev, E. Zahrani, *On the existence of solutions of semilinear impulsive functional differential equations with nonlocal conditions*, 1-13.
- (2) D. Beltiță, *On Banach-Lie algebras, spectral decompositions and complex polarizations*, 13-39.
- (3) T. Brânzar, D. Păunescu, *Commuting triples of subnormal operators and related moment problems*, 39-61.
- (4) M. Buneci, *The equality of the reduced and the full C^* -algebras and the amenability of a topological grupoid*, 61-79.
- (5) L. Carrot, *ρ -numerical radius in Banach spaces*, 79-103.
- (6) G. Cassier, H. Mahzouli, E.H. Zerouali, *Generalized Toeplitz operators and cyclic vectors*, 103-123.
- (7) F. Fidaleo, C. Liverani, *Statistical properties of disordered quantum systems*, 123-143.
- (8) P. Gașpar, *On operator periodically correlated random fields*, 143-157.

- (9) R. Kumar, J.R. Partington, *Weighted composition operators on Hardy and Bergman spaces*, 157-169.
- (10) M. Martin, P. Szeptycki, *Integral Transforms controlled by maximal functions*, 169-185.
- (11) M. Megan, A.L. Sasu, B. Sasu, *Uniform exponential dichotomy and admissibility for linear skew-product semiflows*, 185-197.
- (12) R. Negrea, *On a class of stochastic integral operators of McShane type*, 197-211.
- (13) A.S. Pechentsov, *Regularized traces of differential operators*, 211-229.
- (14) F. Rădulescu, *Irreducible subfactors derived from Popa's construction for non-tracial states*, 229-249.
- (15) M. Shchukin, E. Vatkina, *The structure of some C^* -algebras generated by N idempotents*, 249-255.
- (16) A. Tikhonov, *Transfer functions for "curved" conservative systems*, 255-265.
- (17) N. Tița, *On the distance between an operator and an ideal*, 265-271.
- (18) J.-L. Tu, *The Gamma element for groups which admit a uniform embedding into Hilbert space*, 271-287.
- (19) V.M. Ungureanu, *Uniform exponential stability and uniform observability of time-varying linear stochastic systems in Hilbert spaces*, 287-307.
- (20) G. Weiss, *$B(H)$ -commutators: a historical survey*, 307-321.
- (21) L. Zielinski, *Semiclassical Weyl formula for elliptic operators with non-smooth coefficients*, 321-344.

This book is really first rate and the Publisher, Birkhäuser Verlag, has done a fine job.

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12. Contributions to Current Challenges in Mathematical Fluid Mechanics

by Giovanni P. Galdi, John G. Heywood, Rolf Rannacher, Editors, *Advances in Mathematical Fluid Mechanics*, Birkhäuser Verlag, Basel - Boston - Berlin, viii+152 pp., 2004, ISBN 3 - 7643 - 7104 - 8.

This book knits harmoniously five articles of high quality and originality, each being concerned to a significant topic in the mathematical theory of the Navier-Stokes equations, for compressible and incompressible fluids, and to related questions.

In the first article, "*On Multidimensional Burgers Type Equations with Small Viscosity*" the author, Andrei Biryuk considers the Cauchy problem for a multidimensional Burgers type equation with periodic boundary conditions and deduces upper and lower bounds for derivatives of solutions to this equation in terms of powers of the viscosity and discuss how these estimates relate to the Kolmogorov-Obukov spectral law; next, these estimates obtained are used to get certain bounds for derivatives of solutions of the Navier-Stokes system.

The second article, "*On the Global Well-posedness and Stability of the Navier-Stokes and the Related Equations*", by Dongho Chae and Jihoon Lee, leads with the study of the problem of global well-posedness and stability in the scale invariant Besov spaces for the modified 3D Navier-Stokes equations with the dissipation term, $-\Delta u$ replaced by $(-\Delta)^\alpha u$, $0 \leq \alpha < 5/4$.

In the third article, “*The Commutation Error of the Space Averaged Navier-Stokes Equations on a Bounded Domain*” the authors, A. Dunca, V. John and W.J. Layton prove that the commutation error (which occurs in Large Eddy Simulation of turbulent flows, when the Navier-Stokes equations are convolved with a filter and differentiation and convolution are interchanged) is asymptotically negligible in $L^p(\mathbb{R}^d)$ if and only if the fluid and the boundary exert exactly zero force on each other; in addition they show that the commutation error tends to zero in $H^{-1}(\Omega)$ as the averaging radius $\delta \rightarrow 0$. The convergence is proven also for a weak form of the commutation error.

The fourth article, “*The Nonstationary Stokes and Navier-Stokes Flows Through an Aperture*”, by Toshiaki Hishida leads with the nonstationary Stokes and Navier-Stokes flows in aperture domains $\Omega \subset \mathbb{R}^n$, $n \geq 3$. The author develops the $L^q - L^p$ estimates of the Stokes semigroup and apply them to the Navier-Stokes initial value problem. He obtains the global existence of a unique strong solution, which satisfies the vanishing flux conditions through the aperture and some sharp decay properties as $t \rightarrow \infty$, when the initial velocity is sufficiently small in L^n space.

The latter article, “*Asymptotic Behavior at Infinity of Exterior Three-dimensional Steady Compressible Flow*”, by T. Leonavičienė and K. Pileckas presents steady compressible Navier-Stokes equations with zero velocity conditions at infinity, in a three-dimensional exterior domain. The case of small perturbations of Large potential forces is considered. The results on existence, uniqueness, and asymptotics for the linearized problem and for the nonlinear problem are proved.

This book is addressed to a wide audience in the mathematical and engineering sciences.

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"Stochastic" means being or having a random variable. A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. The random variation is usually based on fluctuations observed in historical data for a selected period using standard time-series techniques. Distributions of potential outcomes are derived from a large number of simulations (stochastic projections) which reflect the random variation in the The financial markets use stochastic models to represent the seemingly random behaviour of assets such as stocks, commodities, relative currency prices (i.e., the price of one currency compared to that of another, such as the price of US Dollar compared to that of the Euro), and interest rates. These models are then used by quantitative analysts to value options on stock prices, bond prices, and on interest rates, see Markov models. Moreover, it is at the heart of the insurance industry.Â In music, mathematical processes based on probability can generate stochastic elements. Stochastic processes may be used in music to compose a fixed piece or may be produced in performance. Stochastic music was pioneered by Iannis Xenakis, who coined the term stochastic music.