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Eductive Stability in Sequential Exchange Economies: An Introduction

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Eductive Stability in Sequential Exchange Economies : An Introduction

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Abstract

This paper provides a survey of results concerning "eductive stability" of equilibrium in an abstract 2-period exchange economy. "Eductive stability" is based on the Common Knowledge considerations underlying the work reported in the book "Assessing rational expectations : eductive stability in economics" (MIT Press, forthcoming) of which this paper is a chapter. "Iteratively Expectational (IE) stability" is a necessary condition of "Eductive stability" : conditions for IE stability as well as for local "Eductive stability" are provided and discussed : these conditions stress a key role for the income effects of savings decisions as well as for their sensitivity to relative prices to-morrow.

Résumé : Ce texte fournit une revue de résultats concernant la "stabilité divinatoire" des équilibres de prévision parfaite dans une économie d'échanges abstraite à deux périodes. Le concept de "stabilité divinatoire" s'appuie sur les considérations de Connaissance Commune qui fondent les travaux exposés dans l'ouvrage "Assessing Rational Expectations : eductive stabilty in economics", dont ce texte constitue l'un des chapitres. La stabilité Itérative (IE-stability) est une condition nécessaire à la stabilité divinatoire. Des conditions pour l'une et l'autre sont fournies et discutées. Ces conditions soulignent le rôle clé des effets-revenus des décisions d'épargne ainsi que celui de leur sensibilité aux prix relatifs de demain.

1 Introduction

This paper provides a brief introduction to the study of eductive stability of sequential equilibria of an exchange economy. The analysis takes place in a deterministic two-period economy in which the allocation of commodities at each period is made on walrasian spot markets. A financial market opens at period 1, allowing to wealth to be transferred between the two periods. Expectational coordination bears upon the second-period prices. Here, as elsewhere in the book, we wonder whether the agents can "educe" the equilibrium: the model, or the aggregate excess demand it generates, as well as rationality, are Common Knowledge (CK). A hypothetical CK restriction triggers collective reasonings that either invalidate it or confirm it and starts a guessing process that converges to the equilibrium.

In contrast to what happens in the production economies envisaged in the previous chapters, the coordination problem is not dominated by the interplay of strategic complementarities and substitutabilities. Both the sensitivity of savings decisions to future prices and the income effects of the savings decisions as well, play a major role on the analysis.

The paper has an introductory purpose. It presents the model and introduces two concepts of expectational stability (part 2). It then derives for these two concepts the formal conditions of success, (part 3) without providing a full picture of the results that might be derived from these premises. Some of these results are, however, evoked in conclusion, together with the relevant references.

2 The economy

2.1 The model

Consider a two-period ($t = 1, 2$) exchange economy, with L (≥ 2) commodities per period, without uncertainty. It is populated by a large set of individuals $i \in \mathbf{I}$ characterized by separable preferences across periods that admit a differentiable, strictly concave per-period representation $U_t^i : \mathbb{R}_+^L \to \mathbb{R}$ and by a strictly positive per-period endowment $\omega_t^i \in \mathbb{R}_{++}^L$. The economy is denoted $\mathbf{\Xi} = \left\{(U_1^i, U_2^i), (\omega_1^i, \omega_2^i), i \in \mathbf{I}\right\}$

At period $t = 1$, spot markets for the L commodities and a financial market, on which a real asset is exchanged, open. They clear, at some equilibrium prices $(p_1, q) \in \mathbb{R}_{++}^L \times \mathbb{R}_{++}$. Then at period $t = 2$, spot markets for the L commodities open, the real asset pays off and some equilibrium prices $p_2 \in \mathbb{R}_{++}^L$ clear the markets. The real asset payoff is normalized to be one unit of the second-period numeraire.

Then, with straightforward notation, an individual facing (p_1, q) today and expecting p_2 to-morrow, solves :

$$\max_{x^i \in \mathbb{R}_+^{2L}, \theta^i \in \mathbb{R}} \left\{ U_1^i(x_1^i) + U_2^i(x_2^i) \right\}$$
$$s.t. \left\{ \begin{array}{c} p_1 x_1^i + q\theta^i \leq p_1 \omega_1^i \\ p_2 x_2^i \leq p_2 \omega_2^i + \theta^i \end{array} \right|$$

Given the sequential nature of trading in periods and the time separability of preferences, the individual problem can be equivalently solved in two stages. First resolve the individual intraperiod consumption problem, denoting by R_t the household income in period t:

$$f_t^i(p_t, R_t) = \arg\max_{x^i \in \mathbb{R}_+^L} \left\{ U_t^i(x_t^i) \middle| \, p_t x_t^i \leq R_t \right\}, t = 1, 2, i \in \mathbf{I}.$$

Then resolve for the individual financial decision θ^i using the per period indirect utility function $U_t^i \left[f_t^i(p_t, R_t) \right]$:

$$\theta^i(p_1, q, p_2) = \arg\max_{\theta^i \in \mathbb{R}} \left\{ U_1^i \left[f_1^i(p_1, p_1\omega_1^i - q\theta^i) \right] + U_2^i \left[f_2^i(p_2, p_2\omega_2^i + \theta^i) \right] \right\}.$$

Households' individual excess demands are then given by

$$\begin{array}{l} z_1^i(p_1, q, p_2) = f_1^i(p_1, p_1\omega_1^i - q\theta^i(p_1, q, p_2)) - \omega_1^i. \\ \theta^i(p_1, q, p_2). \\ z_2^i(p_2, \theta^i(p_1, q, p_2)) = f_2^i(p_2, p_2\omega_2^i + \theta^i(p_1, q, p_2)) - \omega_2^i. \end{array}$$

Definition 1: *A (sequential)* ***perfect-foresight equilibrium***[1] *of allocations* $\left(f_1^{i*}, \theta^{i*}, f_2^{i*}\right)_{i \in \mathbf{I}}$ *and prices* (p_1^*, q^*, p_2^*) *obtain as a solution of the following market clearing equations (MC):*

$$\begin{array}{l} \sum_{i \in \mathbf{I}} z_1^i(p_1^*, q^*, p_2^*) = \sum_{i \in \mathbf{I}} \left[f_1^i(p_1^*, p_1^*\omega_1^i - q^*\theta^i(p_1^*, q^*, p_2^*)) - \omega_1^i \right] = 0 \\ \sum_{i \in \mathbf{I}} \theta^i(p_1^*, q^*, p_2^*) = 0 \\ \sum_{i \in \mathbf{I}} z_2^i(p_2^*, \theta^i(p_1^*, q^*, p_2^*)) = \sum_{i \in \mathbf{I}} \left[f_2^i(p_2^*, p_2^*\omega_2^i + \theta^i(p_1^*, q^*, p_2^*)) - \omega_2^i \right] = 0 \end{array} \tag{MC}$$

This definition does not depend either on the normalization of prices (p_1, q) for the markets of period 1 or on the normalization of prices (p_2) for the spot markets of period 2. It makes sense, since the asset is a real asset that pays in one good, to take this good, by definition good 1, as the numéraire of period 2: this will be done from here on. We will make more specific assumptions on the normalization at the first period, adopting later those that are more convenient for our computations.

[1] It is an equilibrium in plans, prices and price expectations in Radner's sense. Under no uncertainty, it is also called a perfect-foresight equilibrium.

Since we study the conditions under which households can learn second-period market-clearing prices, additional assumptions are needed to guarantee that the perfect-foresight equilibrium of this sequential economy depends continuously on parameters. We shall assume that the equilibrium under consideration is sequentially regular. For that, let us call ∂Z^* the Jacobian matrix of the market clearing equations evaluated at the market clearing prices $(p_1^*, q^*, p_2^*) \in \mathbb{R}_{++}^{2L+1}$,∂Z^* is:

$$\begin{aligned}\partial Z^* &= \begin{pmatrix} \partial_{p_1}\left[\sum z_1^{i*}\right] & \partial_q\left[\sum z_1^{i*}\right] & \partial_{p_2}\left[\sum z_1^{i*}\right] \\ \partial_{p_1}\left[\sum \theta^{i*}\right] & \partial_q\left[\sum \theta^{i*}\right] & \partial_{p_2}\left[\sum \theta^{i*}\right] \\ \partial_{p_1}\left[\sum z_2^{i*}\right] & \partial_q\left[\sum z_2^{i*}\right] & \partial_{p_2}\left[\sum z_2^{i*}\right] \end{pmatrix} \\ &= \begin{pmatrix} \partial_{(p_1,q)}\begin{bmatrix} \sum z_1^{i*} \\ \sum \theta^{i*} \end{bmatrix} & \partial_{p_2}\begin{bmatrix} \sum z_1^{i*} \\ \sum \theta^{i*} \end{bmatrix} \\ \partial_{(p_1,q)}\left[\sum z_2^{i*}\right] & \partial_{p_2}\left[\sum z_2^{i*}\right] \end{pmatrix}\end{aligned}$$

where $\partial_p\left[\sum z_t^i\right]_{(p_1^*,q^*,p_2^*)} \equiv \partial_p\left[\sum z_t^{i*}\right]$ and $\partial_p\left[\sum \theta^i\right]_{(p_1^*,q^*,p_2^*)} \equiv \partial_p\left[\sum \theta^{i*}\right]$.

Denote $\partial_{p_2}\left[\sum z_2^{i*}(\theta^i)\right] = \sum \partial_{p_2} z_2^{i*}(\theta^i)$ as the partial price derivative of the second-period spot commodity excess demand holding the savings' decision fixed. Then $\sum \partial_{p_2} z_2^{i*}(\theta^i) = \partial_{p_2}\left[\sum z_2^{i*}\right] - \sum \partial_R f_2^{i*} \frac{\partial \theta^{i*}}{\partial p_2}$.

Definition 2: *A perfect foresight equilibrium is* ***sequentially regular*** *if* $\left(\partial_{(p_1,q)}\begin{bmatrix} \sum z_1^{i*} \\ \sum \theta^{i*} \end{bmatrix}\right)$ is of rank L *and* $\left(\partial_{p_2}\left[\sum z_2^{i*}(\theta^i)\right]\right)$ is of rank $L-1$.

2.2 The Expectational Stability criteria

Let us make clear the institutional arrangements under which the market works : at the first period, every household submits demand functions both on the spot markets and on the financial market; at the second period, every household will similarly submit demand functions for the second-period spot markets. The questions of computation or implementation of the flexible price equilibrium are then solved and can be ignored.

Now the central question, the standard question concerning "eductive stability" as analysed in this book can be described as follows. Assume a tentative CK restriction of the following kind: the first- (respectively second-) period market clearing prices lie in some non trivial neighbourhood around the equilibrium (resp. $p_2' \in V(p_2^*)$. "Eductive stability" means that there exists such a non trivial local neighbourhood, the Common Knowledge of which triggers CK of the equilibrium.

We focus here on two necessary conditions of "eductive stability", as just sketched. For dealing with the first one, consider individuals' point expectations[2] about second-period market-clearing prices $p_2^{e,i}$ lying in a set $P_2^0 \subset$

[2] With an abuse of notation, we will adopt the convention that $p_2^{e,i}$ denotes both the vector of households' individual expectations and a given component of that vector.

$\mathbb{R}_{++}^{I(L-1)}$. Individuals submit excess-demand functions of the form $z_1^i(p_1, q, p_2^{e,i}), \theta^i(p_1, q, p_2^{e,i})$ and $z_2^i(p_2, \theta^i(p_1', q', p_2^{e,i}))$ such that for any fixed assignment of expectations $p_2^{e,i}, \forall i : \mathbf{I} \to P_2^0$, (p_1', q') is a period 1 price equilibrium:

$$\begin{aligned}\sum_{i \in \mathbf{I}} z_1^i(p_1', q', p_2^{e,i}) &= 0 \\ \sum_{i \in \mathbf{I}} \theta^i(p_1', q', p_2^{e,i}) &= 0\end{aligned}$$

Denote by $E_1(p_2^{e,i})$ the set of such first-period price equilibria. If P_2^0 is small enough, from sequential regularity and the implicit function theorem, and for any given normalization of first-period prices, there exists a function $\varphi_1 : P_2^0 \to E_1(p_2^{e,i})$ mapping any given assignment of expectations on second-period market-clearing prices $p_2^{e,i}, \forall i$ to first-period equilibrium prices (p_1', q'), that is $(p_1', q') = \varphi_1(p_2^{e,i})$. Note that $(p_1^*, q^*) = \varphi_1(p_2^*)$.

For a given pair of first-period prices (p_1, q), let p_2'' denote a period 2 price equilibrium when households hold their savings decisions on the basis of a given assignment of second-period price expectations $p_2^{e,i}$ satisfying:

$$\sum_{i \in \mathbf{I}} z_2^i(p_2'', \theta^i(p_1, q, p_2^{e,i})) = 0$$

Denote by $E_2(p_1, q, p_2^{e,i})$ the set of such second-period price equilibria. Again, by sequential regularity and the implicit function theorem, it follows that there exists a function $\varphi_2 : \mathbb{R}_{++}^L \times P_2^0 \to E_2(p_1, q, p_2^{e,i})$ mapping any given assignment of expectations on second-period market-clearing prices $p_2^{e,i}, \forall i$ and (normalized) parametric first-period prices (p_1, q) to second-period equilibrium prices, that is $p_2'' = \varphi_2(p_1, q, p_2^{e,i})$. In particular, note that $p_2^* = \varphi_2(p_1^*, q^*, p_2^*)$.

Finally, denote by $\psi_t(.)$ the restriction of the functions $\varphi_t(.)$ just defined, whenever households expectations on second period market clearing prices coincide $p_2^{e,i} = p_2^e$:

$$\varphi_t(.)|_{p_2^{e,i} = p_2^e} = \psi_t(.)$$

Now expectational stability is defined as follows:

Definition 3: *The perfect foresight price equilibrium is said to be (locally) Iteratively Expectationally Stable, **IE-Stable** if and only if there exists a neighborhood V of the second period market clearing price p_2^* such that $\forall p_2^e \in V(p_2^*)$, the system*

$$p_2^e(t+1) = \psi_2\left[\psi_1(p_2^e(t)), p_2^e(t)\right]$$

converges to p_2^*.

This is the standard idea of Iterative Expectational stability that has been found several times through the book: if agents have a homogenous belief p_2^e on the second-period price equilibrium, then first-period prices will be $\psi_1(p_2^e)$ and the actual second-period price equilibrium will be $\psi_2 [\psi_1(p_2^e), p_2^e]$. Definition 3 defines the virtual dynamics: perceived prices at t give actual prices at $t+1$.

Now allowing for households expectations to differ across, we can formulate the following definition :

Definition 4: *The perfect foresight price equilibrium is said to be (weakly) locally* ***Eductively Stable (W-ES)*** *if and only if there exists a small neighborhood V of the second-period market-clearing price p_2^* such that $\forall p_2^{e,i} \in V(p_2^*)$,*

$$\varphi_2 \left[\varphi_1(p_2^{e,i}), p_2^{e,i} \right] \subset V(p_2^*).$$

This definition is in the spirit of the eductive stability idea developed in this book: heterogenous (point-) expectations in V leave the system strictly in V. Hence, along the lines of the analysis of others chapters, it is plausible, although not formally established here, that some additional assumptions on the behavior of agents under uncertainty, (expected-utility maximization in particular) imply that W-ES as defined here is equivalent to "eductive stability" in the sense of this book : CK that (stochastic) expectations have their support in V triggers CK that $p_2 = p_2^*$.

The readers will also check, unsurprisingly, since we now allow for heterogenous expectations, that the ES criterion defined here is more demanding than IE stability. They will also remark that although the precise functional form of the functions $\varphi_t(.)$ may depend on the normalization chosen, the properties stressed in defintions 3 and 4 are unaffected by the normalization convention. Convergence of the dynamical system in the definitions of expectational stability is unaffected.

$>$From here on, we take the convenient first-period price normalization $q = 1$; the price of the "real asset" is 1.

3 Conditions for Expectational Stability

With this normalization, the market clearing equations can be writtenas follows :

$$\begin{array}{l} Z_1(p_1^*, p_2^*) = \sum_{i \in \mathbf{I}} z_1^i(p_1^*, p_2^*)) = \sum_{i \in \mathbf{I}} \left[f_1^i(p_1^*, p_1^* \omega_1^i - \theta^i(p_1^*, p_2^*)) - \omega_1^i \right] = 0 \\ Z_2(p_1^*, p_2^*) = \sum_{i \in \mathbf{I}} z_2^i(p_1^*, p_2^*)) = \sum_{i \in \mathbf{I}} \left[f_2^i(p_2^*, p_2^* \omega_2^i + \theta^i(p_1^*, p_2^*)) - \omega_2^i \right] = 0 \end{array}$$

From here, we shall compute a first order linear approximation of mappings $\varphi_2 [\varphi_1(.), (.)]$ and $\psi_2 [\psi_1(.)), (.)]$ around the equilibrium $*$.We proceed as follows.With straightforward notation:

$$d\varphi_2 [\varphi_1(.), (.)] = \sum_{i \in \mathbf{I}} [(\partial \varphi_{21})(\partial \varphi_{1i}) + (\partial \varphi_{22i})] dp_2^{e,i}$$

Using the above definitions, the above derivatives can be made explicit. It is left to the reader to convince himself, from inspection of the relationships among Z_i, z_1^i, z_2^i, f_1^i, and f_2^i that

$$(\partial\varphi_{21}) = -\left[\partial_{p_2} Z_2^* - \underbrace{\sum_{i\in\mathbf{I}} \partial_R f_2^{i*} \frac{\partial\theta^i}{\partial p_2}}_{\equiv M}\right]^{-1} (\partial_{p_2} Z_2^*)$$

$$(\partial\varphi_{1i}) = -\left(\partial_{p_1} Z_1^*\right)^{-1} (\partial_{p_2} z_1^i)$$

$$(\partial\varphi_{22i}) = -\left[\partial_{p_2} Z_2^* - \underbrace{\sum_{i\in\mathbf{I}} \partial_R f_2^{i*} \frac{\partial\theta^i}{\partial p_2}}_{\equiv M}\right]^{-1} (\partial_R f_2^{i*} \tfrac{\partial\theta^i}{\partial p_2})$$

so that:

$$d\varphi_2\left[\varphi_1(.), (.)\right] = \left[\partial_{p_2} Z_2^* - M\right]^{-1} \left[\sum_{i\in\mathbf{I}} \left\{(\partial_{p_2} Z_2^*)\left(\partial_{p_1} Z_1^*\right)^{-1}(\partial_{p_2} z_1^i) - (\partial_R f_2^{i*} \frac{\partial\theta^i}{\partial p_2})\right\} dp_2^{e,i}\right]$$

or

$$d\varphi_2\left[\varphi_1(.), (.)\right] = \sum_{i\in\mathbf{I}} \left(\Phi^i dp_2^{e,i}\right)$$

with

$$\Phi^i = \left[\partial_{p_2} Z_2^* - M\right]^{-1} \left\{(\partial_{p_2} Z_2^*)\left(\partial_{p_1} Z_1^*\right)^{-1}(\partial_{p_2} z_1^i) - (\partial_R f_2^{i*} \frac{\partial\theta^i}{\partial p_2})\right\}$$

Equivalently:

$$\Phi^i = \left\{\left[\partial_{p_2} Z_2^* - M\right]^{-1} \left[(\partial_{p_2} Z_2^*)\left(\partial_{p_1} Z_1^*\right)^{-1} \partial_R f_1^{i*} - \partial_R f_2^{i*}\right] \frac{\partial\theta^i}{\partial p_2}\right\}$$

In case of homogenous expectations, the computation simplifies and

$$d\psi_2\left[\psi_1(.)), (.)\right] = \left[\partial_{p_2} Z_2^* - \underbrace{\sum_{i\in\mathbf{I}} \partial_R f_2^{i*} \frac{\partial\theta^i}{\partial p_2}}_{\equiv M}\right]^{-1} \left[(\partial_{p_1} Z_2^*)\left(\partial_{p_1} Z_1^*\right)^{-1}(\partial_{p_2} Z_1^*) - \underbrace{\sum_{i\in\mathbf{I}} \partial_R f_2^{i*} \frac{\partial\theta^i}{\partial p_2}}_{\equiv M}\right] dp_2^e$$

a formula that is obtained in Guesnerie and Hens (2000) and can also be rewritten as

$$d\psi_2\left[\psi_1(.),(.)\right]=\sum_{i\in\mathbf{I}}\left\{\left[\partial_{p_2}Z_2^*-M\right]^{-1}\left[\left(\partial_{p_1}Z_2^*\right)\left(\partial_{p_1}Z_1^*\right)^{-1}\partial_R f_1^{i*}-\partial_R f_2^{i*}\right]\frac{\partial\theta^i}{\partial p_2}\right\}dp_2^e$$

Or:

$$d\psi_2\left[\psi_1(.)),(.)\right]=\left(\sum_{i\in\mathbf{I}}\Phi^i\right)dp_2^e$$

Relying on previous discussions in this book, one sees that:

- local IE stability amounts to the fact that the eigenvalue of highest modulus of $\partial\psi_2\left[\psi_1(.),(.)\right]$ has modulus smaller than 1.

- or, if the matrix $\sum\Phi^i$ is semisimple, and if n denotes the matrix norm induced by the Euclidean norm on the basis of its eigenvectors, then local IE-stability amounts to:

$$n(\sum\Phi^i)<1$$

- the condition:

$$\sum n(\Phi^i)<1$$

is strictly more demanding, but ensures that our condition of ES stability holds.

- in fact this condition is a sufficient condition for ES stability whatever the choice of the matrix norm made here.

The main findings can be summarized as follows:

Proposition 1 *1 - A necessary and sufficient condition of (local) IE-stability is : The modulus of the eigenvalue of highest modulus of* $\left[\partial_{p_2}Z_2^*-M\right]^{-1}\left[\left(\partial_{p_1}Z_2^*\right)\left(\partial_{p_1}Z_1^*\right)^{-1}\left(\partial_{p_2}Z_1^*\right)-M\right]$ *(where* $M=\sum_{i\in\mathbf{I}}\partial_R f_2^{i*}\frac{\partial\theta^i}{\partial p_2}$*)*

is smaller than 1.

2- (local W) ES stability $\rightarrow$*(local) IE stability*

3 -A sufficient condition for (local W) ES stability is $\sum n(\Phi^i)<1$*, where* n *is a matrix norm and* $\Phi^i=\left\{\left[\partial_{p_2}Z_2-M\right]^{-1}\left[\left(\partial_{p_1}Z_2\right)\left(\partial_{p_1}Z_1\right)^{-1}\partial_R f_1^i-\partial_R f_2^i\right]\frac{\partial\theta^i}{\partial p_2}\right\}$

It is left to the reader to comment the formulas more lengthily.

4 Summary and Conclusions

The above conditions may be exploited in order to derive more explicit and economically intuitve conditions, as undertaken in the (now) unpublished papers of Ghosal(2001) and Guesnerie-Hens (2001).

We make only a few remarks :

- It is easily seen in the above stability conditions, that if savings are insensitive to second period spot prices, then both IE-stability and ES stability hold (since $\frac{\partial \theta^i}{\partial p_2}$ is then small).

-It is also straightforward, although less transparent from the conditions, that if all agents have almost identical homothetical preferences in period 2, then the income effects of savings decisisons do not significantly affect the second period excess demand, so that the equilibrium is stable for both criteria.

- Finding more general conditions that weaken savings sensitivity or attenuate the effect on demand of second-period savings induced income effects is not straightforward. In particular, standard conditions on excess demand, such as gross substitutability for example, do not seem powerful[3].

The understanding of the exchange economy case is however a prerequisite to the understanding of the whole general equilibrium expectational coordination problem. The brief introduction presented here provides then a first and hopefully useful complement to the analysis of the first two chapters of this part.

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[3] Gross substitutability makes the algorithm of Balasko (1994) converge. This Gauss-Seidl-like algorithm computes equilibrium discounted prices successively on the Arrow-Debreu markets of the two periods. The conceptual inspiration of the exercise is quite different (equilibrium prices in this setting are not equilibrium prices of our sequential model). However, the superficial proximityof the formulas defining the dynamics in both cases might lead one to believe that insights could be gained in our case from the mentioned results. This does not seem to be the case. In particular, the simulations of Guesnerie-Hens (2000), as well as their comparison of mathematical conditions, make clear that the two convergence problems are unconnected.