

## Relativistic Cosmology

---

Cosmology has been transformed by dramatic progress in high-precision observations and theoretical modelling. This book surveys key developments and open issues for graduate students and researchers. Using a relativistic geometric approach, it focuses on the general concepts and relations that underpin the standard model of the Universe.

Part 1 covers foundations of relativistic cosmology, whilst Part 2 develops the dynamical and observational relations for all models of the Universe based on general relativity. Part 3 focuses on the standard model of cosmology, including inflation, dark matter, dark energy, perturbation theory, the cosmic microwave background, structure formation and gravitational lensing. It also examines modified gravity and inhomogeneity as possible alternatives to dark energy. Anisotropic and inhomogeneous models are described in Part 4, and Part 5 reviews deeper issues, such as quantum cosmology, the start of the universe and the multiverse proposal. Colour versions of some figures are available at [www.cambridge.org/9780521381154](http://www.cambridge.org/9780521381154).

**George F. R. Ellis** FRS is Professor Emeritus at the University of Cape Town, South Africa. He is co-author with Stephen Hawking of *The Large Scale Structure of Space-Time*.

**Roy Maartens** holds an SKA Research Chair at the University of the Western Cape, South Africa, and is Professor of Cosmology at the University of Portsmouth, UK.

**Malcolm A. H. MacCallum** is Director of the Heilbronn Institute at Bristol, and is President of the International Society on General Relativity and Gravitation.

Cambridge University Press  
978-0-521-38115-4 - Relativistic Cosmology  
George F. R. Ellis, Roy Maartens and Malcolm A. H. Maccallum  
Frontmatter  
[More information](#)

---

Cambridge University Press  
978-0-521-38115-4 - Relativistic Cosmology  
George F. R. Ellis, Roy Maartens and Malcolm A. H. Maccallum  
Frontmatter  
[More information](#)

---

# Relativistic Cosmology

---

GEORGE F. R. ELLIS

University of Cape Town

ROY MAARTENS

University of Portsmouth and University of the Western Cape

MALCOLM A. H. MACCALLUM

University of Bristol



CAMBRIDGE  
UNIVERSITY PRESS

Cambridge University Press  
978-0-521-38115-4 - Relativistic Cosmology  
George F. R. Ellis, Roy Maartens and Malcolm A. H. MacCallum  
Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521381154](http://www.cambridge.org/9780521381154)

© Cambridge University Press 2012

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without  
the written permission of Cambridge University Press.

First published 2012

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Ellis, George F. R. (George Francis Rayner)  
Relativistic cosmology / George Ellis, Roy Maartens, Malcolm MacCallum.  
p. cm.

Includes bibliographical references and index.

ISBN 978-0-521-38115-4

1. Cosmology. 2. Relativistic astrophysics. 3. Relativistic quantum theory.  
I. Maartens, R. (Roy) II. MacCallum, M. A. H. III. Title.

QB981.E4654 2012

523.1-dc23 2011040518

ISBN 978-0-521-38115-4 Hardback

Additional resources for this publication at [www.cambridge.org/9780521381154](http://www.cambridge.org/9780521381154).

Cambridge University Press has no responsibility for the persistence or  
accuracy of URLs for external or third-party internet websites referred to  
in this publication, and does not guarantee that any content on such  
websites is, or will remain, accurate or appropriate.

# Contents

*Preface* *page xi*

## Part 1 Foundations

<b>1</b>	<b>The nature of cosmology</b>	<b>3</b>
1.1	The aims of cosmology	3
1.2	Observational evidence and its limitations	5
1.3	A summary of current observations	9
1.4	Cosmological concepts	17
1.5	Cosmological models	20
1.6	Overview	23
<b>2</b>	<b>Geometry</b>	<b>25</b>
2.1	Manifolds	26
2.2	Tangent vectors and 1-forms	28
2.3	Tensors	31
2.4	Lie derivatives	34
2.5	Connections and covariant derivatives	35
2.6	The curvature tensor	37
2.7	Riemannian geometry	39
2.8	General bases and tetrads	51
2.9	Hypersurfaces	53
<b>3</b>	<b>Classical physics and gravity</b>	<b>56</b>
3.1	Equivalence principles, gravity and local physics	56
3.2	Conservation equations	61
3.3	The field equations in relativity and their structure	64
3.4	Relation to Newtonian theory	69

## Part 2 Relativistic cosmological models

<b>4</b>	<b>Kinematics of cosmological models</b>	<b>73</b>
4.1	Comoving coordinates	73
4.2	The fundamental 4-velocity	74
4.3	Time derivatives and the acceleration vector	75
4.4	Projection to give three-dimensional relations	76

4.5	Relative position and velocity	79
4.6	The kinematic quantities	80
4.7	Curvature and the Ricci identities for the 4-velocity	86
4.8	Identities for the projected covariant derivatives	88
<b>5</b>	<b>Matter in the universe</b>	<b>89</b>
5.1	Conservation laws	90
5.2	Fluids	95
5.3	Multiple fluids	101
5.4	Kinetic theory	104
5.5	Electromagnetic fields	110
5.6	Scalar fields	115
5.7	Quantum field theory	117
<b>6</b>	<b>Dynamics of cosmological models</b>	<b>119</b>
6.1	The Raychaudhuri–Ehlers equation	119
6.2	Vorticity conservation	124
6.3	The other Einstein field equations	126
6.4	The Weyl tensor and the Bianchi identities	132
6.5	The orthonormal 1+3 tetrad equations	134
6.6	Structure of the 1+3 system of equations	139
6.7	Global structure and singularities	143
6.8	Newtonian models and Newtonian limits	147
<b>7</b>	<b>Observations in cosmological models</b>	<b>153</b>
7.1	Geometrical optics and null geodesics	153
7.2	Redshifts	156
7.3	Geometry of null geodesics and images	159
7.4	Radiation energy and flux	161
7.5	Specific intensity and apparent brightness	167
7.6	Number counts	170
7.7	Selection and detection issues	171
7.8	Background radiation	172
7.9	Causal and visual horizons	173
<b>8</b>	<b>Light-cone approach to relativistic cosmology</b>	<b>180</b>
8.1	Model-based approach	180
8.2	Direct observational cosmology	181
8.3	Ideal cosmography	186
8.4	Field equations: determining the geometry	187
8.5	Isotropic and partially isotropic observations	190
8.6	Implications and opportunities	194

### Part 3 The standard model and extensions

<b>9 Homogeneous FLRW universes</b>	201
9.1 FLRW geometries	202
9.2 FLRW dynamics	210
9.3 FLRW dynamics with barotropic fluids	212
9.4 Phase planes	220
9.5 Kinetic solutions	225
9.6 Thermal history and contents of the universe	226
9.7 Inflation	238
9.8 Origin of FLRW geometry	246
9.9 Newtonian case	247
<b>10 Perturbations of FLRW universes</b>	249
10.1 The gauge problem in cosmology	250
10.2 Metric-based perturbation theory	251
10.3 Covariant nonlinear perturbations	262
10.4 Covariant linear perturbations	267
<b>11 The cosmic background radiation</b>	282
11.1 The CMB and spatial homogeneity: nonlinear analysis	282
11.2 Linearized analysis of distribution multipoles	287
11.3 Temperature anisotropies in the CMB	292
11.4 Thomson scattering	294
11.5 Scalar perturbations	295
11.6 CMB polarization	300
11.7 Vector and tensor perturbations	303
11.8 Other background radiation	303
<b>12 Structure formation and gravitational lensing</b>	307
12.1 Correlation functions and power spectra	307
12.2 Primordial perturbations from inflation	309
12.3 Growth of density perturbations	317
12.4 Gravitational lensing	330
12.5 Cosmological applications of lensing	339
<b>13 Confronting the Standard Model with observations</b>	345
13.1 Observational basis for FLRW models	346
13.2 FLRW observations: probing the background evolution	351
13.3 Almost FLRW observations: probing structure formation	355
13.4 Constraints and consistency checks	363
13.5 Concordance model and further issues	366

<b>14 Acceleration from dark energy or modified gravity</b>	370
14.1 Overview of the problem	370
14.2 Dark energy in an FLRW background	373
14.3 Modified gravity in a RW background	376
14.4 Constraining effective theories	390
14.5 Conclusion	391
<b>15 'Acceleration' from large-scale inhomogeneity?</b>	395
15.1 Lemaître–Tolman–Bondi universes	395
15.2 Observables and source evolution	399
15.3 Can we fit area distance and number count observations?	401
15.4 Testing background LTB with SNIa and CMB distances	403
15.5 Perturbations of LTB	406
15.6 Observational tests of spatial homogeneity	411
15.7 Conclusion: status of the Copernican Principle	415
<b>16 'Acceleration' from small-scale inhomogeneity?</b>	416
16.1 Different scale descriptions	416
16.2 Cosmological backreaction	421
16.3 Specific models: almost FLRW	423
16.4 Inhomogeneous models	426
16.5 Importance of backreaction effects?	432
16.6 Effects on observations	435
16.7 Combination of effects: altering cosmic concordance?	440
16.8 Entropy and coarse-graining	441
<b>Part 4 Anisotropic and inhomogeneous models</b>	
<b>17 The space of cosmological models</b>	447
17.1 Cosmological models with symmetries	447
17.2 The equivalence problem in cosmology	452
17.3 The space of models and the role of symmetric models	453
<b>18 Spatially homogeneous anisotropic models</b>	456
18.1 Kantowski–Sachs universes: geometry and dynamics	457
18.2 Bianchi I universes: geometry and dynamics	458
18.3 Bianchi geometries and their field equations	462
18.4 Bianchi universe dynamics	467
18.5 Evolution of particular Bianchi models	474
18.6 Cosmological consequences	481
18.7 The Bianchi degrees of freedom	486

<b>19 Inhomogeneous models</b>	488
19.1 LTB revisited	490
19.2 Swiss cheese revisited	491
19.3 Self-similar models	493
19.4 Models with a $G_3$ acting on $S_2$	495
19.5 $G_2$ cosmologies	496
19.6 The Szekeres–Szafron family	498
19.7 The Stephani–Barnes family	501
19.8 Silent universes	501
19.9 General dynamics of inhomogeneous models	502
19.10 Cosmological applications	503
<b>Part 5 Broader perspectives</b>	
<b>20 Quantum gravity and the start of the universe</b>	511
20.1 Is there a quantum gravity epoch?	511
20.2 Quantum gravity effects	512
20.3 String theory and cosmology	516
20.4 Loop quantum gravity and cosmology	526
20.5 Physics horizon	530
20.6 Explaining the universe – the question of origins	532
<b>21 Cosmology in a larger setting</b>	535
21.1 Local physics and cosmology	535
21.2 Varying ‘constants’	539
21.3 Anthropic question: fine-tuning for life	542
21.4 Special or general? Probable or improbable?	546
21.5 Possible existence of multiverses	548
21.6 Why is the universe as it is?	554
<b>22 Conclusion: our picture of the universe</b>	555
22.1 A coherent view?	555
22.2 Testing alternatives: probing the possibilities	558
22.3 Limits of cosmology	559
<b>Appendix Some useful formulae</b>	561
A.1 Constants and units	561
A.2 1+3 covariant equations	563
A.3 Frequently used acronyms	565
<i>References</i>	566
<i>Index</i>	606

Cambridge University Press  
978-0-521-38115-4 - Relativistic Cosmology  
George F. R. Ellis, Roy Maartens and Malcolm A. H. Maccallum  
Frontmatter  
[More information](#)

---

## Preface

This book provides a survey of modern cosmology emphasizing the relativistic approach. It is shaped by a number of guiding principles.

- **Adopt a geometric approach** Cosmology is crucially based in spacetime geometry, because the dominant force shaping the universe is gravity; and the best classical theory of gravity we have is Einstein's general theory of relativity, which is at heart a geometric theory. One should therefore explore the spacetime geometry of cosmological models as a key feature of cosmology.
- **Move from general to special** One can best understand the rather special models most used in cosmology by understanding relationships which hold in general, in all spacetimes, rather than by only considering special high symmetry cases. The properties of these solutions are then seen as specialized cases of general relations.
- **Explore geometric as well as matter degrees of freedom** As well as exploring matter degrees of freedom in cosmology, one should examine the geometric degrees of freedom. This applies in particular in examining the possible explanations of the apparent acceleration of the expansion of the universe in recent times.
- **Determine exact properties and solutions where possible** Because of the nonlinearity of the Einstein field equations, approximate solutions may omit important aspects of what occurs in the full theory. Realistic solutions will necessarily involve approximation methods, but we aim where possible to develop exact relations that are true generically, on the one hand, and exact solutions of the field equations that are of cosmological interest, on the other.
- **Explore the degree of generality or speciality of models** A key theme in recent cosmological writing is the idea of 'fine tuning', and it is typically taken to be bad if a universe model is rather special. One can, however, only explore the degree of speciality of specific models by embedding them in a larger context of geometrically and physically more general models.
- **Clearly relate theory to testability** Because of the special nature of cosmology, theory runs into the limits of the possibility of observational testing. One should therefore pursue all possible observational consistency checks, and be wary of claiming theories as scientific when they may not in principle be testable observationally.
- **Focus on physical and cosmological relevance** The physics proposed should be plausibly integrated into the rest of physics, where it is not directly testable; and the cosmological models proposed should be observationally testable, and be relevant to the astronomical situation we see around us.

- **Search for enduring rather than ephemeral aspects** We have attempted to focus on issues that appear to be of more fundamental importance, and therefore will not fade away, but will continue to be of importance in cosmological studies in the long term, as opposed to ephemeral topics that come and go.

Part 1 presents the foundations of relativistic cosmology. Part 2 is a comprehensive discussion of the dynamical and observational relations that are valid in all models of the universe based on general relativity. In particular, we analyse to what extent the geometry of spacetime can be determined from observations on the past light-cone. The standard Friedmann–Lemaître–Robertson–Walker (FLRW) universes are discussed in depth in Part 3, covering both the background and perturbed models. We present the theory of perturbations in both the standard coordinate-based and the 1+3 covariant approaches, and then apply the theory to inflation, the cosmic microwave background, structure formation and gravitational lensing. We review the key unsolved issue of the apparent acceleration of the expansion of the universe, covering dark energy models and modified gravity models. Then we look at alternative explanations in terms of large scale inhomogeneity or small scale inhomogeneity.

Anisotropic homogeneous (Gödel, Kantowski-Sachs and Bianchi) and inhomogeneous universes (including the Szekeres models) are the focus of Part 4, giving the larger context of the family of possible models that contains the standard FLRW models as a special case. In all cases the relation of the models to astronomical observations is a central feature of the presentation.

The text concludes in Part 5 with a brief review of some of the deeper issues underlying all cosmological models. This includes quantum gravity and the start of the universe, the relation between local physics and cosmology, why the universe is so special that it allows intelligent life to exist, and the issue of testability of proposals such as the multiverse.

The text is at an advanced level; it presumes a basic knowledge of general relativity (e.g. as in the recent introductory texts of Carroll (2004), Stephani (2004), Hobson, Efstathiou and Lasenby (2006) and Schutz (2009)) and of the broad nature of cosmology and cosmological observations (e.g. as in the recent introductory books of Harrison (2000), Ferreira (2007) and Silk (2008)). However, we provide a self-contained, although brief, survey of Riemannian geometry, general relativity and observations.

Our approach is similar to that of our previous reviews, Ellis (1971a, 1973), MacCallum (1973, 1979), Ellis and van Elst (1999a) and Tsagas, Challinor and Maartens (2008), and it builds on foundations laid by Eisenhart (1924), Synge (1937), Heckmann and Schucking (1962), Ehlers (1961), Trümper (1962, and unpublished), Hawking (1966) and Kristian and Sachs (1966). This approach differs from the approach in the excellent recent texts by Peacock (1999), Dodelson (2003), Mukhanov (2005), Weinberg (2008), Durrer (2008), Lyth and Liddle (2009) and Peter and Uzan (2009), in that we emphasize a covariant and geometrical approach to curved spacetimes and where possible consider general geometries instead of restricting considerations to the FLRW geometries that underlie the standard models of cosmology.

A further feature of our presentation is that although it is solidly grounded in relativity theory, we recognize the usefulness of Newtonian cosmological models and calculations. We detail how the Newtonian limit follows from the relativistic theory in situations of cosmological interest, and make clear when Newtonian calculations give a good approximation to the results of the relativistic theory and when they do not.

It is not possible to cover all of modern cosmology in depth in one book. We present a summary of present cosmological observations and of modern astrophysical understanding of cosmology, drawing out their implications for the theoretical models of the universe, but we often refer to other texts for in-depth coverage of particular topics. We are relatively complete in the theory of relativistic cosmological models, but even here the literature is so vast that we are obliged to refer to other texts for fuller details. In particular, the very extensive discussions of spatially homogeneous cosmologies and of inhomogeneous cosmologies in the books by Wainwright and Ellis (1997), Krasinski (1997), and Bolejko *et al.* (2010) complement and extend our much shorter summaries of those topics in Part 4. Our guiding aim is to present a coherent core of theory that is not too ephemeral, i.e. that in our opinion will remain significant even when some present theories and observations have fallen away. Only the passage of time will tell how good our judgement has been.

We have given numerical values for the key cosmological parameters, but these should be interpreted only as indicative approximations. The values and their error bars change as observations develop, so that no book can give definitive values. Furthermore, there are inherent limitations to parameter values and error bars – which depend on the particular observations used, on the assumptions made in reducing the observational data, on the chosen theoretical model needed to interpret the observations, and on the type of statistical analysis used.

In the text we have two kinds of interventions apart from the usual apparatus of footnotes and references: namely, exercises and problems. The *Exercises* enable the reader to develop and test his or her understanding of the main material; we believe we know the answers to all the exercises, or at least where the answer is given in the literature (in which case an appropriate reference is provided). By contrast, the *Problems* are unsolved questions whose solution would be of some interest, or in some cases would be a major contribution to our understanding.

We are grateful to numerous people who have played an important role in developing our understanding of cosmology: we cannot name them all (though most of their names will be found in the reference list at the end), but we would particularly like to thank John Barrow, Bruce Bassett, Hermann Bondi,<sup>1</sup> Marco Bruni, Anthony Challinor, Chris Clarkson, Peter Coles, Rob Crittenden, Peter Dunsby, Ruth Durrer, Jürgen Ehlers,<sup>1</sup> Henk van Elst, Pedro Ferreira, Stephen Hawking, Charles Hellaby, Kazuya Koyama, Julien Larena, David Matravers, Charles Misner, Jeff Murugan, Bob Nichol, Roger Penrose, Felix Pirani, Alan Rendall, Wolfgang Rindler, Tony Rothman, Rainer Sachs, Varun Sahni, Misao Sasaki, Bernd Schmidt, Engelbert Schucking, Dennis Sciama,<sup>1</sup> Stephen Siklos, John Stewart, Bill Stoeger,

<sup>1</sup> deceased

Cambridge University Press  
978-0-521-38115-4 - Relativistic Cosmology  
George F. R. Ellis, Roy Maartens and Malcolm A. H. Maccallum  
Frontmatter  
[More information](#)

---

Reza Tavakol, Manfred Trümper, Christos Tsagas, Jean-Philippe Uzan, John Wainwright and David Wands for insights that have helped shape much of what is presented here. We thank the FRD and NRF (South Africa), the STFC and Royal Society (UK), and our departments, for financial support that has contributed to this work.

George F. R. Ellis  
Roy Maartens  
Malcolm A. H. MacCallum

Cosmology - Cosmology - Relativistic cosmologies: To derive his 1917 cosmological model, Einstein made three assumptions that lay outside the scope of his equations. The first was to suppose that the universe is homogeneous and isotropic in the large (i.e., the same everywhere on average at any instant in time), an assumption that the English astrophysicist Edward A. Milne later elevated to an entire philosophical outlook by naming it the cosmological principle. Relativistic Astrophysics and Cosmology offers a succinct and self-contained treatment of general Physical Foundations of Cosmology. 442 Pages • 2006 • 2.56 MB • 7,486 Downloads. PHYSICAL FOUNDATIONS OF COSMOLOGY Inflationary cosmology has been developed over the last 20 years Physic Relativistic Astrophysics and Cosmology - A Primer. 295 Pages • 2007 • 14.87 MB • 763 Downloads. Table of contents page iii 1. Special Relativity. 10. 1.1. Motivation. 10. 1.2. The postulates General Relativity, Bla View Relativistic cosmology Research Papers on Academia.edu for free. Origen's Cosmology and Ontology of Time constitute a major catalyst and a massive transformation in the development of Christian doctrine. The author challenges the widespread impression about this theology being bowled head over heels by its encounter with Platonism, Gnosticism, or Neoplatonism, and casts new light on Origen's grasp of the relation between Hellenism, Hebrew thought and Christianity.