Minimum Delay Systems for the Modeling of Transmission Lines at EMT Power System Studies

Martin G. Vega, J. L. Naredo, O. Ramos-Leaños

***Abstract*--The Phase-Domain Line Model, or Universal Line Model (ULM), is considered one of the most advanced for time-domain analysis of EMTs in power lines and cables. This model involves two convolution processes that usually are performed through recursions. The recursive processes follow from the rational representations of both matrices, the one of characteristic admittances Yc and the one of propagation H. To attain a rational representation for matrix H, with high accuracy, it is needed to previously identify and extract from it of its modal travel-time factors. The accuracy at determining the modal travel-times to be extracted has a considerable impact on the accuracy of the synthesized rational representation of matrix H, as well as on the accuracy of the transmission-line model. This paper presents a method to determine travel-times that are optimum in the strict sense of Systems Theory. This method is based on determining the main features of systems satisfying the Paley-Wiener criterion.**

***Keywords*: Electromagnetic transients, time delays, Paley-Wiener, ULM, minimum phase, NLT.**

I. Introduction

Traveling-wave-based transmission-line models, such as the Universal Line Model (ULM), can include all the frequency-dependent effects of line and cable parameters [1]. These models involve two convolution processes that usually are performed through recursions. These recursive processes follow from rational representations of the characteristic admittance matrix $\mathbf{Y}_c$ and of the propagation matrix **H** [1].

To attain a rational representation for matrix **H** with high accuracy, it is needed to previously identify and extract from it its modal travel-time factors [1,2]. The accuracy at determining those modal travel-times has a considerable impact on the accuracy of the synthesized rational representation of matrix **H**, as well as on the accuracy of the transmission-line model [2,5]. The standard method for determining the modal travel-times makes use of an integral formula by Bode [6].

Once the travel times are obtained and extracted from the propagation matrix one proceeds to the rational fitting, usually by applying Vector Fitting (VF) [1,2]. The numerical implementation of Bode's formula does not always offer good accuracy. For this reason, it has been proposed elsewhere the use of optimization methods for finding the travel times that lead to the rational fit of **H** with the lowest RMS error. The optimization technique proposed in [3] makes use of Brent's search [17]. Disadvantages of this technique are that each iteration requires applying a full VF process and that the resulting model is not always minimum-phase.

Rather than searching for the travel times that produce the least RMS error, it is proposed in this paper to search for either the one with the minimum phase or the one with the minimum phase deviation. These two alternatives guarantee minimum phase fitted models. In addition, the search for minimum phase does not require the iterative use of VF; thus, its computational efficiency is much higher than that of the one based on minimum RMS error.

It also is proposed in this paper to test, not only for the fitting error of fitted **H**, but also for its error to step response. The use of the step as a test signal is considered appropriate because of its rich spectral contents. Accuracy tests are conducted here using a frequency-domain line model and the Numerical Laplace Technique (NLT) [9]. The NLT is tuned for a numerical relative-error of 10^{-9}. Finally, comparisons are provided between the two proposed criteria and the ones based on Bode's technique and of Brent's optimization with VF. The comparisons involve a single-phase aerial line and a single-phase underground cable.

II. Causal Systems

Physical (realizable) systems must be causal; that is, an effect cannot precede its cause. In the sense of system theory, the physical realizability of transfer-functions is defined by causality. Step responses of causal systems can only exist for $t \geq 0$. The necessary and sufficient condition for a system to be causal is that its transfer function $H(\omega)$ complies with the following relation [8]:

$$\int_{-\infty}^{+\infty} \frac{\ln|H(\omega)|}{1+\omega^2} d\omega < \infty \qquad (1)$$

If, in addition to (1), $|H(\omega)|$ is quadratically integrable, magnitude $|H(\omega)|$ can be associate with a phase response so the resulting $H(\omega)$ is causal. Expression (1) corresponds to the Paley-Wiener criterion. Transfer functions satisfying this criterion have the general form:

$$\mathrm{H}(\mathrm{j}\omega) = \mathrm{H}_{\min}(\mathrm{j}\omega)\mathrm{H}_{\mathrm{ap}}(\mathrm{j}\omega) \qquad (2)$$

where $\mathrm{H}_{\min}(\omega)$ is a minimum phase transfer function and H_{ap} represents an all-pass system. The minimum phase function:

$$\mathrm{H}_{\min}(j\omega) = |\mathrm{H}(\mathrm{j}\omega)|e^{\varphi_{min}} \qquad (3)$$

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has minimum lag φ_{min} and has all its zeros and poles lying on the left half s-plane.

The all-pass transfer-function $H_{ap}(j\omega)$ in (2) has unit magnitude; it thus only affects the phase relations of an input signal [10, 11].

III. Travelling-Wave Line Model

Travelling-wave line models require rational approximations for the characteristic admittance matrix $\mathbf{Y}_c$ and for the propagation matrix $\mathbf{H}$ [1,4]:

$$\mathbf{Y}_c = \sqrt{\mathbf{YZ}}\mathbf{Z}^{-1} = \mathbf{\Gamma}\mathbf{Z}^{-1} \tag{4}$$

$$\mathbf{H} = e^{-\sqrt{\mathbf{YZ}}l} \tag{5}$$

where $\mathbf{Z}$ and $\mathbf{Y}$ are the respective matrices of series impedances and of shunt admittances.

Matrix $\mathbf{Y}_c$ is approximated rationally as follows:

$$\mathbf{Y}_c \cong \mathbf{G}_0 + \sum_{k=1}^{N_y} \frac{\mathbf{G}_k}{s - p_k} \tag{6}$$

where N_y is the order of the rational approximation, p_k is the $k-th$ pole, $\mathbf{G}_k$is the $k-th$ matrix of residues associated to p_k and $\mathbf{G}_0$ is a constant matrix obtained as the limit $s \longrightarrow \infty$ of $\mathbf{Y}_c$. The poles of $\mathbf{Y}_c$ are determined from its matrix trace.

The rational fitting of $\mathbf{H}$ is substantially more involving than that of $\mathbf{Y}_c$. The reason for this is that every element of $\mathbf{H}$ involves all the line modal-delays. Delay separation is thus needed to attain accurate and low order fits. Separation can be obtained through the following factorization [18],

$$\mathbf{H} = \sum_{i=1}^{N} \mathbf{D}_i e^{-\tilde{\gamma}_i l}\, e^{-s\tau_i} = \sum_{i=1}^{N} \widetilde{\mathbf{H}}_i\, e^{-s\tau_i} \tag{7}$$

where N is the number of the line propagation-modes (or of the line independent conductors), matrix $\mathbf{D}_i$ is the idempotent matrix associated to the *i-th* line mode [18], τ_i is the *i-th* delay, $\tilde{\gamma}_i$ is the corresponding propagation modal-constant with the *i-th* delay being extracted, $\widetilde{\mathbf{H}}_i$ is the propagation function for the *i-th* mode without the *i-th* delay-factor. The rational approximation of $\mathbf{H}$ can be expressed as follows:

$$\mathbf{H} \cong \sum_{i=1}^{N} \left[\sum_{k=1}^{N_{h,i}} \frac{\boldsymbol{R}_{i,k}}{s - q_{i,k}} \right] e^{-s\tau_i} \tag{8}$$

where N indicates the number of line propagation-modes (or of line independent-conductors) and $N_{h,i}$ is the order of the rational-approximation for the *i-th* propagation-matrix. Rational approximations usually are obtained with the Vector Fitting (VF) technique [2].

IV. Frequency Domain Line Model

A transmission line can be represented in the frequency domain as a two-port nodal network [13,14]:

$$\begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{V}_L \end{bmatrix} \tag{9}$$

with:

$$\mathbf{A} = (\mathbf{U} - \mathbf{H}^2)^{-1}(\mathbf{H}^2 + \mathbf{U})\mathbf{Y}_c, \tag{10}$$

$$\mathbf{B} = (\mathbf{U} - \mathbf{H}^2)^{-1}(-2\mathbf{H})\mathbf{Y}_c. \tag{11}$$

Transient responses of transmission lines can be obtained in the Laplace domain through from (9). The corresponding time-domain waveforms are obtained through the subsequent application of the Numerical Laplace transform (NLT). A numerical accuracy of 10^{-9} is attained here by employing $N = 2^{19}$ (524 288) samples [9]. NLT results are taken here as reference to evaluate the accuracy of each methodology considered in this paper.

V. Time Delay Identification

Travel times associated to matrix $\mathbf{H}$ can be determined by a technique based on minimum phase identification. The standard technique being used with the ULM to identify travel times is based on Bode's theorem [6]. This theorem establishes the following relationship between the magnitude of an analytic function and its minimum phase function [6]:

$$\varphi_i(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{d(\ln|\mathrm{H}_i|)}{du_i} \ln\left(\coth\frac{|u_i|}{2}\right) du_i \tag{12}$$

where $u = \Omega_i/\omega$ and Ω_i corresponds to a cut-off frequency, typically at $|\mathrm{H}_i| = 0.1|\mathrm{H}_i(\omega)|$. After solving (12), the travel time can be calculated as follows:

$$\tau_i = \frac{l}{v_i(\Omega_i)} + \frac{\varphi_i}{\Omega_i} \tag{13}$$

Here v_i is the velocity of the *i-th* mode and l is the transmission line-length.

The actual application of (12) must be done numerically and this causes the introduction of errors in the estimated travel-times. Because of these errors, the delay factors cannot be extracted completely from $\mathbf{H}$ and the rational synthesis becomes inaccurate. It thus proposed in [3] to overcome this problem by applying Brent's search to determine the travel times that result in the fitted $\mathbf{H}$ with the minimum RMS-error.

As it has been previously mentioned, the problems with the search method based on Brent's optimization are that each iteration involves the application of VF and that the fitted form of $\mathbf{H}$ often is of non-minimum phase. An alternative is proposed here consisting in the search for the travel time that minimizes the following expression for phase deviation:

$$dev\phi_{\tau_k} = \sum_{m=1}^{N_\omega} \left|\varphi_{\tau_k}(m)\right|^2 \tag{17}$$

where N_ω is the number of frequency samples of $\mathbf{H}$, $\varphi_{\tau_k}(m)$ represents the *m-th* sample of the phase characteristic for the *k-th* time delay τ_k. The criterion to choose the next τ_k is as in [17]. This process guarantees the minimum-phase characteristic for the fitted part of $\mathbf{H}$ and it only requires one application of VF. Even though this method is simple, it can fail if the bracketing interval is not appropriate.

Another alternative for determining minimum phase condition is based on the following ratio:

$$\rho_\tau = \left|\sum_{k=1}^{N_z} Re\{Z_k\}\right| \Bigg/ \sum_{k=1}^{N_z} |Re\{Z_k\}| \;;\; \begin{cases} \rho = 1; & \text{minimum phase} \\ \rho \neq 1; & \text{non minimum phase} \end{cases} \tag{18}$$

where N_z is the number of zeros (Z_k) of the fitted part of **H**. Expression (18) is combined with a bisection method to determine the time-delays that result in minimum phase rational approximations ($\rho = 1$). The number of time delays satisfying the minimum phase condition is large; it is proposed here that the smallest one is to be selected.

VI. Test Cases

A. Single-phase aerial line

Consider the single-phase overhead transmission line shown in Fig. 1 along with its additional data provided in Table I. The line is connected at $x = 0$ to an ideal voltage source that injects a voltage unit step. At the far end the line is open-ended. The waveform has been obtained with the Numerical Laplace Transform using 2^{19} samples to guarantee an accuracy of 10^{-9}.

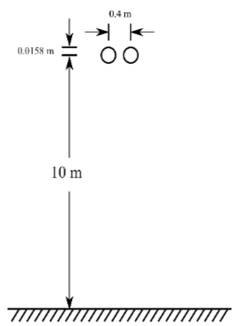

Fig. 1. Single phase overhead line.

TABLE I
Line Data

Line length	30 km
Conductors resistivity	3.21e-8 $\Omega \cdot m$
Ground resistivity	100 $\Omega \cdot m$
Ground relative permittivity	1

Figure. 2 provides the plot of ρ_τ given by (18) versus travel-time for the line of Fig.1. Recalling that $\rho_\tau = 1$ implies minimum phase condition, the plot shows that this condition is not unique. Figure 2 shows also the travel-times determined through the methods being considered here: Bode's integral, Brent's iterations with VF, minimum phase deviation (17) and minimum-phase ratio (18).

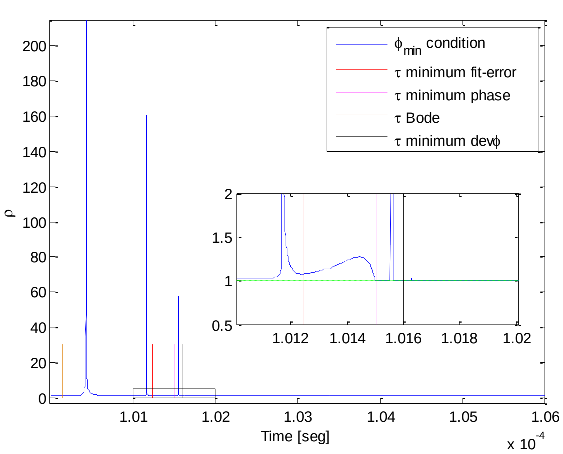

Fig. 2. Plot of ρ_τ versus travel-time.

Each one of the travel-times being obtained with the four techniques being considered here is now used first to extract the delay-factor from **H** and then to apply VF with 10 poles. Table II provides the RMS-errors obtained for each case.

TABLE II
Accuracy of Delay Identification Methods

Method	Time Delay	Fit error
Minimum RMS-fiterror	1.012422032538708E-04	1.214327540033741E-05
Minimum phase deviation	1.016000134612205E-04	2.728885014864769E-05
Bode's Integral	1.001426314054519E-04	2.443542200601405E-05
Minimum Phase	1.015010000000000E-04	1.591003303296070E-05

Fig. 3 shows the transient waveform of voltage at $x = l$ as obtained with the NLT (reference) and with the ULM employing the four travel-times at Table II and their corresponding fitted H-functions. The differences among the plots in Fig. 3 cannot be seen by eye. Figure 4 provides their relative-error plots taking as reference the NLT waveform. Note that the lowest error at the transient response is obtained when the travel-time is estimated with the minimum phase-deviation criterion. Note also from Table II that not necessarily the minimum RMS-error in the fitting of **H** implies minimum error at the transient response.

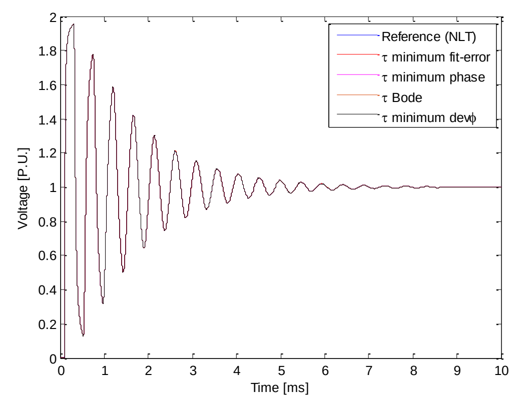

Fig. 3. Transient response

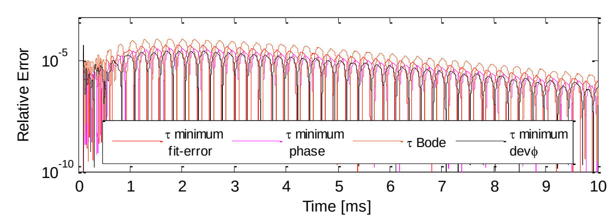

Fig. 4. Relative error plots.

B. Underground cable, core response

Consider a 5 km single-core underground cable with its transversal geometry shown in Fig. 5. The table III provides the cable data required for its ULM representation. In this case, **H** is a 2×2 matrix with two propagation modes: a coaxial mode with travel time τ_c and a ground-return mode with travel time τ_s. Table IV provides the estimates for these travel-times as obtained with the four methods being considered here. The table also provides the corresponding RMS fitting-errors.

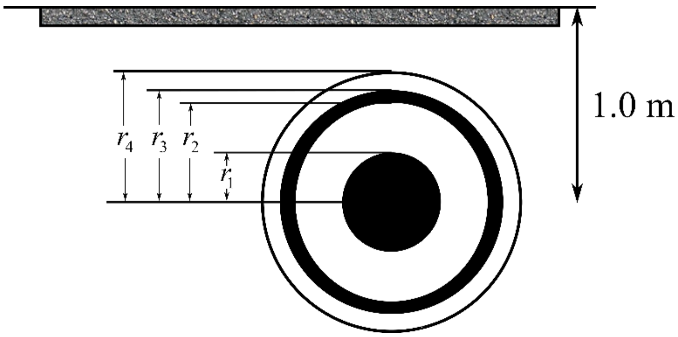

Fig. 5 Underground cable

TABLE III
CABLE DATA

Item	Property
Core	$r_1 = 39\ mm, \rho_c = 3.365\text{E-}08\ \Omega \cdot m$
Insulation	$r_2 = 57.25\ mm, \varepsilon_r = 2.85$
Sheath	$r_3 = 57.47\ mm, \rho_s = 1.718\text{E-}08\ \Omega \cdot m$
Jacket	$r_4 = 62\ mm, \varepsilon_r = 2.51$

TABLE IV
ACCURACY OF DELAY IDENTIFICATION METHODS

Method	Time Delay	Fit Error
Minimum RMS-fit error	τ_c=2.777400505173475E-05	2.388896814564825E-04
	τ_s=2.218543471451478E-04	3.315760607061955E-04
Minimum phase deviation	τ_c=2.823403392873278E-05	2.645481088729894E-02
	τ_s=2.147232833833232E-04	5.177308878427851E-04
Bode's Integral	τ_c=2.797759291385173E-05	2.778796375087252E-04
	τ_s=2.224495660472671E-04	3.433906986012019E-04
Minimum Phase	τ_c=2.811158333333333E-05	1.161222758578107E-03
	τ_s=2.246533333333333E-04	7.368437356373795E-04

Consider that the cable of Fig. 5 is connected as in Fig. 6, with its sheath solidly grounded at both ends. At $x = 0$, a unit step of voltage is injected into the core through an ideal source and at $x = l$ the core is open-ended. Figure 7 provides the voltage responses at $x = l$ being obtained with the NLT technique (reference) and with the ULM employing the four travel-time sets at Table IV along with their corresponding fitted H matrix-functions.

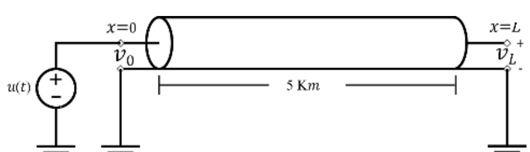

Fig. 6 Step core voltage with sheath grounded.

The differences among the plots in Fig. 7 can now be seen by eye. Nevertheless, their differences are provided in the form of relative error plots at Fig. 8. The plot obtained with the NLT has been taken as the reference. Note that this time the lowest error at the transient response is obtained when travel-times are estimated with Brent´s search and VF.

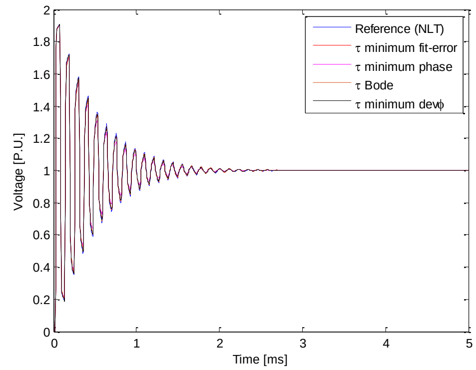

Fig. 7 Core voltage.

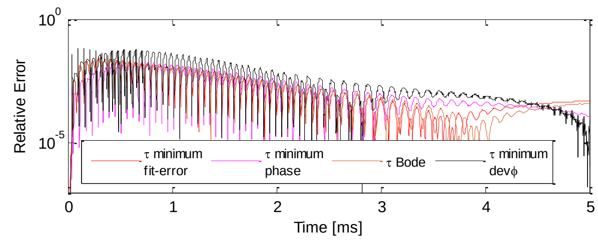

Fig. 8 Relative-error plots.

C. *Underground cable, sheath responses*

The cable of Fig. 5 is connected now as in Fig. 9. This time both, the core and the sheath are open-ended at the remote end ($x = l$). As an ideal voltage soured injects a unit step at the core near end ($x = 0$), the voltage waveform responses at the core far end are as in Fig. 10, while the sheath induced voltage waveforms are as is Fig. 12. All these waveforms have been obtained with the NLT (reference) and with the ULM employing the four travel-time sets at Table IV. Figure 11 provides the relative error plots corresponding to the results in Fig.10, while Fig. 13 provides the error plots for Fig. 12 results.

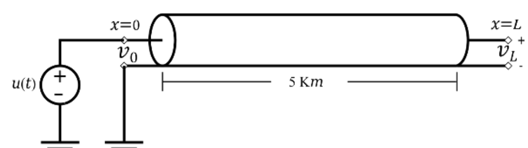

Fig. 9. Step voltage excitation on core.

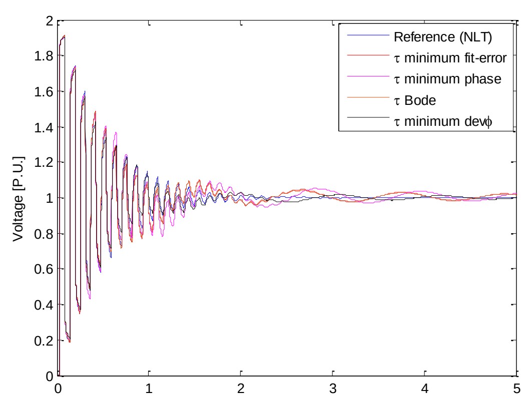

Fig. 10. Core voltage

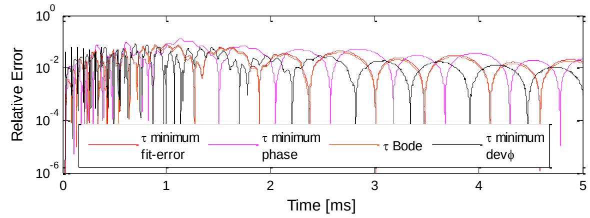

Fig. 11. Relative-error plots.

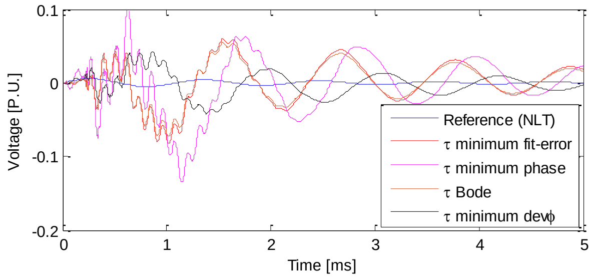

Fig. 12. Induced sheath voltages.

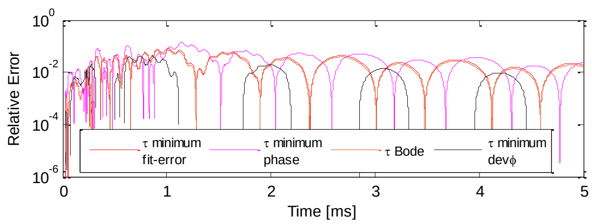

Fig. 13. Relative-error plots.

D. *Underground cable, core induced voltages*

Consider now that Fig, 5 cable is connected as in Fig. 14. This time the voltage ideal source injecting a unit step is connected to the cable sheath at the near end ($x = 0$). At this same end, the cable core is grounded. At the far end ($x = l$) both, the core and the sheath are open-ended. Figure 15 provides the induced voltage-waveforms at the core far-end. Figure 16 provides, as before, the relative-error waveforms corresponding to the plots of Fig. 15 and taking the one obtained with the NLT as reference. Finally, Fig. 17 provides the plots of the sheath transient responses and Fig. 18 the associated relative-error plots.

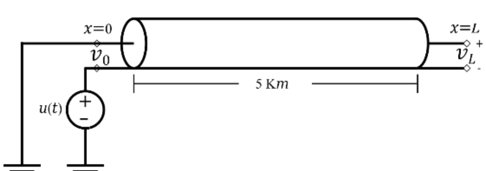

Fig. 14. Step voltage excitation of cable sheath.

VII. Discussion

The previous test-cases corroborate that the process of travel-time identification has an impact on the accuracy of traveling-wave line models. The RMS error at the rational fit of $\mathbf{H}$ can be used as a search criterion. Nevertheless, the minimum error of the fit does not guarantee the minimum error at the transient response. Another search criteria are the travel-times that, when extracted from $\mathbf{H}$, result in a minimum phase or in a

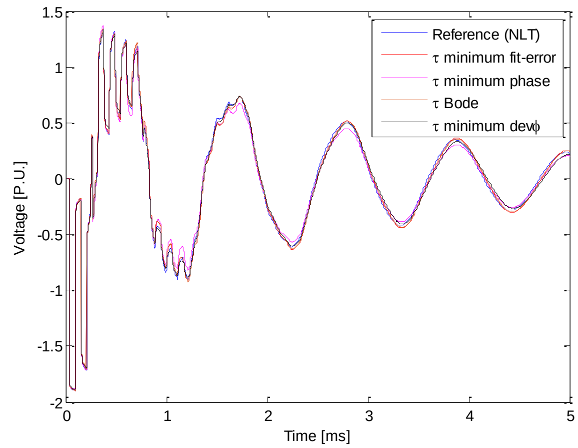

Fig. 15. Induced core voltage

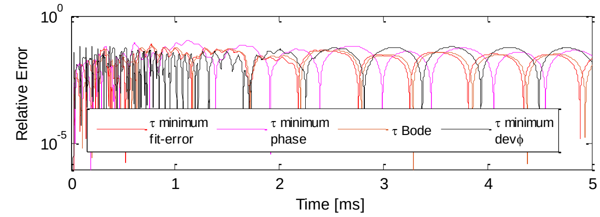

Fig. 16. Relative-error plots.

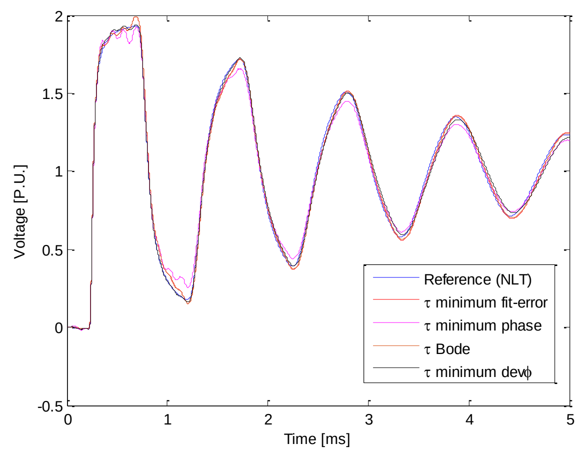

Fig. 17. Sheath voltage.

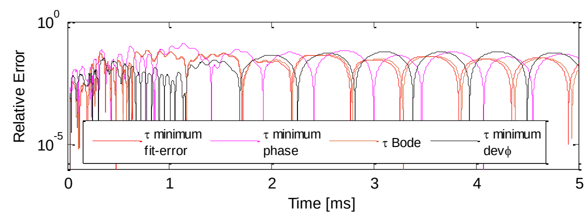

Fig. 18. Relative-error plots.

minimum phase deviation model. In a strict sense, a physical lumped-element system, such as $\mathbf{Y}_c$, must be a minimum phase one; a physical distributed-element system, like $\mathbf{H}$, should consist of a minimum phase part cascaded with a maximum phase (or all-pass) one. Transient-response comparisons among the four criteria do not show a clear winner. These can be seen at Tables 2 and 4, as well as at Fig. 3. Nevertheless, in favor of the two criteria being proposed here, one can say that these two guarantee the model synthesis with minimum phase;

in addition, the minimum phase deviation method requires much less computations than the one that uses Brent's search and VF.

VIII. Conclusions

The research being reported in this paper focuses on the effective estimation of travel-times for multi-conductor power lines and cables, as well as on the impact of these estimates on the accuracy of the traveling-wave models for the lines in question. An adequate travel-time estimation may require the use of iterative optimizations. It has been contended here that the optimization method proposed in [3] results computationally costly, since each iteration requires a complete rational fit using VF. It has also been pointed out that this method does not guarantee a minimum phase synthesis. The two methods being introduced in this paper have a much lower computational cost and the fitted models are minimum phase. The results in this paper show that fitting-error minimization does not imply minimum errors at the transient response of the synthesized line models. Simulation tests have shown that in some cases the method proposed in [3] results in slightly lower errors than those of the methods being proposed here and, in some others, this is the other way around. In conclusion, there is not a clear winner regarding the accuracy at transient responses.

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Abstract--The Phase-Domain Line Model, or Universal Line Model (ULM), is considered one of the most advanced for time-domain analysis of EMTs in power lines and cables. This model involves two convolution processes that usually are performed through recursions.Â raveling-wave-based transmission-line models, such as the Universal Line Model (ULM), can include all the frequency-dependent effects of line and cable parameters [1]. These models involve two convolution processes that usually are performed through recursions. These recursive processes follow from rational representations of the characteristic admittance matrix ð˜ð’„ and of the propagation matrix ð‡ [1]. Lines at EMT Power System Studies. Martin G. Vega, J. L. Naredo, O. Ramos-LeaÃ±os. Abstract--The Phase-Domain Line Model, or Universal Line. Model (ULM), is considered one of the most advanced for time-. domain analysis of EMTs in power lines and cables. This model. involves two convolution processes that usually are performed. through recursions. The recursive processes follow from the. rational representations of both matrices, the one of. characteristic admittances Yc and the one of propagation H. To.