

EXPERIENCING GEOMETRY

EUCLIDEAN AND NON-EUCLIDEAN
WITH HISTORY

THIRD EDITION

David W. Henderson

Daina Taimiņa

Cornell University, Ithaca, New York



Upper Saddle River, New Jersey 07458

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Geometry is defined as the experience of a person's field of vision becoming partially or completely encompassed by fast-moving, colorful, and indescribably complex geometric patterns, [1][2][3][4][5][6][7] form constants, [8][9] shapes, [4] fractals, [4] and colors. These geometric forms can also become structured and organized in a manner that appears to present genuine information to the person experiencing them far beyond the perception of meaningless, although complex, shapes, and colors. The geometric representations may Experiencing Geometry. Average rating: 0 out of 5 stars, based on 0 reviews Write a review. This button opens a dialog that displays additional images for this product with the option to zoom in or out. Tell us if something is incorrect. Experiencing Geometry. Average rating: 0 out of 5 stars, based on 0 reviews Write a review. \$107.80 \$107.80 \$107.80 \$107.80. Manufacturers, suppliers and others provide what you see here, and we have not verified it. See our disclaimer. 9780131437487 Experiencing Geometry. Specifications. Publisher.

