Spacecraft Collision Probability

F. Kenneth Chan

The Aerospace Press • El Segundo, California

American Institute of Aeronautics and Astronautics, Inc. • Reston, Virginia

Chapter 1: Overview

1.1 Introduction

The exploitation of the near-Earth space environment by commercial, military, and scientific interests during the last 50 years has led to the increasing likelihood of collisions between orbiting objects. Because their positions are not known to extremely high precision, the inevitable question of collision probability becomes one of great interest. In such an encounter, one of the orbiting objects is usually an operational spacecraft and is referred to as the primary object. The other, referred to as the secondary object, can be an active, inactive, or even a rogue spacecraft; it may be a piece of orbiting debris of natural or man-made origin.

The timely publication of this book serves the needs of the space community by exploring the subject of collision probability. This first chapter summarizes the topics that appear in the remainder of the book, including subjects such as different types of encounters; the extent of encounter regions and transit times; probabilities of collisions for a variety of orbits and circumstances, including encounters with multiple objects; and evasive maneuvers. This chapter introduces the concepts and analyses, the problems and solutions that will be developed more rigorously and comprehensively in the later chapters. Each section that follows corresponds to a different chapter.

In order to compute the probability of collision, we need to formulate the problem rigorously within the framework of probability theory. If we know the probability density functions (pdf) describing the uncertainties of the positions of the two objects, we can determine the probability that they are within a specified volume. If we assume that the random variables associated with these pdfs are independent (or less stringently uncorrelated), then we can take the product of these pdfs to obtain the joint pdf. We can then integrate over the product space of random variables that corresponds to the specified volume of interest. Even for simple pdfs, this approach is rather involved.

Another approach is to consider the relative position of one object with respect to the other and obtain the pdf describing the uncertainty of their relative positions. We can then integrate over the region of space swept out by the volume of interest as one object moves relative to the other. This is the approach taken in this book.

Until around 1990, most analyses[1.1-1.5] on spacecraft collision probability were concerned with a spacecraft going through a layer of orbiting space debris. They have been based on the Poisson distribution (i.e., they modeled the process as in the kinetic theory of gases, in which the gas molecules move along straight lines with random directions and their number density is statistically uniform). The general applicability of this model is questionable because the orbiting objects move in essentially Keplerian orbits and their number density varies from the equator to their inclination latitude (the highest latitude reached during an orbit), where the statistical density is several orders of magnitude higher.

Most present analyses[1.6-1.14] on spacecraft collision probability are concerned with close encounters between two orbiting objects. They are based on the Gaussian distribution because the primary spacecraft and secondary space objects can be tracked and their positions determined to within the errors associated with the corresponding position covariance ellipsoids. The collision probability is then computed using this information.

1.2 Spacecraft Encounters

Chapter 2 addresses the mathematical and physical formulation of an encounter between two orbiting objects. If we assume that the positional uncertainties of the two objects are uncorrelated, we may simply add the two covariances to obtain the combined covariance of the relative position. To facilitate the formulation of the collision probability computation, an encounter coordinate system is defined at the instant of “nominal” closest approach. In this system, the origin is at the secondary object, the y-axis is in the direction of the relative velocity, the x-axis is in the direction of the primary, and the z-axis completes the right-handed triad. The (x, z)-plane is defined as the encounter plane. This definition of the encounter system is applicable to the case of most spacecraft in low Earth orbits (LEOs) with large relative velocities engaged in short-term encounters whereby they experience a flyby lasting for only a few seconds. It is also true that in such cases the relative motion is essentially rectilinear in the encounter region.

Of special significance is the unstable behavior of such an encounter coordinate system when the relative velocity is small (sometimes a few meters per second). Because the relative velocity can change in direction, the coordinate axes so defined change direction drastically and the elements of the combined covariance fluctuate violently. This radical behavior precludes the encounter system from being used for long-term encounters because they are characterized by small relative velocities and nonrectilinear relative motion in the encounter region. This is the case with some pairs of spacecraft, located in geosynchronous orbits (GEOs), that can spend close to a day in the vicinity of each other.

Thus, two different methods must be used to study these two significantly different problems. Hitherto, analyses on collision probabilities have assumed that the encounters are of the short-term nature without explicitly stating as much. This book formulates the theory and the algorithms for computing the collision probabilities for these two cases and also presents results illustrating the salient differences between them.

1.3 The Encounter Region

Chapter 3 explains how the extent of the encounter region and the transit time are determined. For any object orbiting Earth, we know that the trajectory is curved as a result of the effects of gravitational forces. Thus, in general, the relative motion of one object with respect to another will be a curved line. However, under certain circumstances, it may be unnecessary to account for the details of the nonlinear relative motion when computing the probability of collision between them. Thus, in the vicinity of the point of closest approach for the case of short-term encounters, we assume that the two objects move along straight lines so that their relative motion is also described by a straight line. The question is: When can we make this assumption? Stated in another way, what is the extent of this region for which the orbits can be considered to be straight lines? It is important to determine the extent of the encounter region.

In computing the probability of collision, we need to integrate over the region of random variables that corresponds to a collision. For simplicity, we assume that both the objects are spherical. Let r_p and r_s respectively denote the radius of the primary and the secondary object. Then, if the secondary comes within a sphere with radius $r_A = r_p + r_s$ centered at the primary, there will be a physical overlap between them.

We have stated that we base the collision probability integral on the relative position covariance. It is a three-dimensional integral that can be time-consuming to evaluate. If we make the assumption of rectilinear relative motion, then the volume swept out by the sphere of radius r_A is a long cylinder extending along the y-direction from $-\infty$ to $+\infty$. The outcome of this process is that, instead of having to deal with a cumbersome three-dimensional pdf, we now consider the corresponding marginal two-dimensional pdf. The volume integration over an infinitely long circular cylinder is replaced by the area integration over a circular cross section. Let σ be the standard deviation along the relative velocity direction. If we perform numerical analysis, we obtain a requirement of 17σ for the approximating straight-line path to achieve a 17-digit accuracy and a requirement of 6σ for the approximating straight-line path to achieve a 2-digit accuracy. These lengths are used to estimate the extent of the encounter region for which the relative motion may be regarded as rectilinear.

We can also obtain the time to traverse the encounter region. For large angles of approach, the transit time is relatively small so that the approximation becomes valid. However, for small approach angles, the relative velocity of transit is very small and the transit time is very large. Consequently, the rectilinear relative motion approximation is invalid and it is not possible to reduce the three-dimensional collision integral to a two-dimensional one.

1.4 The Isotropic Problem

Even after transitioning to a two-dimensional integral, we find that there is no known closed-form solution when the integrand is a bivariate Gaussian pdf, that is, the two standard deviations σ_x and σ_z are not equal. However, for the isotropic Gaussian pdf for which $\sigma_x = \sigma_z \equiv \sigma$, we can transform the two-dimensional integral exactly to a one-dimensional integral involving the Rician pdf as the integrand. Chapter 4 examines this isotropic case.

The Rician pdf arises frequently in the detection of signals in the presence of noise. It also arises in target detection by pulsed radar, noncoherent detection of band-passed binary signals, accuracy of unguided ballistic bombs hitting their targets, and artillery effectiveness of fragmentation bombs. Now, we find that it arises in spacecraft collision probability for the special isotropic case.

It is possible to derive an analytical expression for the Rician integral in the form of an infinite series that is convergent for all values of the input collision parameters. This infinite series yields the zeroth-order and first-order approximations for the collision probability by retaining respectively one or two terms in the series. The Rician integral forms an important topic of this book.

For simplicity, we shall take σ to be between 1 km and 10 km for LEOs. For most spacecraft dimensions, we can safely take the radius r_A up to 100 m. (Most large spacecraft are at most 40 m and the International Space Station is approximately 100 m at the largest dimension, so that r_A is, in these cases, respectively 20 m and 50 m.) For encounters between two orbiting objects, we can take the miss distance x_e up to 100 km, which is more than we really need to consider. Hence, we can establish the following present limits for our collision parameters:

$$0 < \left(\frac{r_A}{\sigma}\right) \leq 10^{-1} \qquad \text{and} \qquad 0 \leq \left(\frac{x_e}{\sigma}\right) \leq 100 \tag{1.1}$$

For some future systems, we may have very accurate orbit determinations and thus σ may lie between 10 m and 100 m. Accordingly, we may consider miss distance up to 100 m. Therefore, the future limits become

$$0 < \left(\frac{r_A}{\sigma}\right) \leq 10 \qquad \text{and} \qquad 0 \leq \left(\frac{x_e}{\sigma}\right) \leq 10 \tag{1.2}$$

We find that the first three orders of approximation yield sufficient accuracy for the present range of collision parameters but that substantially higher orders are needed for the anticipated future range of parameters. Specifically, for most present-day applications (for which the combined object radius is approximately less than one-tenth the standard deviation), the zeroth-order and first-order approximations provide collision probabilities accurate to within 1% error.

1.5 Analytical Expressions for Short-Term Encounters

Whereas Chapter 4 considers an isotropic Gaussian pdf in the encounter (x, z)-plane, even though we might have started with a three-dimensional covariance that is not necessarily isotropic, Chapter 5 considers the nonisotropic case for which the two standard deviations σ_x and σ_z are not equal in the two-dimensional

Gaussian pdf used in computing the collision probability. The covariance C, in general, contains an off-diagonal term with a nonzero correlation coefficient ρ_{xz}.

To handle this case, we rotate the (x, z)-coordinate axes to new (x', z')-coordinate axes so that the new covariance C' has only diagonal terms $\sigma_{x'}^2$ and $\sigma_{z'}^2$. We then transform the (x', z')-coordinate axes to new (x'', z'')-coordinate axes so that the new covariance C'' has equal diagonal terms $\sigma_{x''}^2$ and $\sigma_{z''}^2$. In this process, the circular cross section becomes an elliptical cross-sectional area of integration.

Next, if we approximate this elliptical cross section by an equivalent circular one having the same area and the same centroid, then we convert the probability problem into the isotropic one already solved. Thus, we are able to use all the previous results of the isotropic problem already available for the collision probability. The approximation in representing the elliptical cross section by a circular cross section ultimately determines the accuracy of using the Rician pdf in our modeling. The requirement is basically that the combined object radius r_A is approximately less than one-tenth the standard deviation $\sigma \equiv \sqrt{(\sigma_{x'}\sigma_{z'})}$ and the encounter plane elliptic aspect ratio $1 \leq (\sigma_{x'}/\sigma_{z'}) \leq 10$.

To obtain a measure of accuracy, we compare this formulation (Chan) with three other well-known formulations for use in computing spacecraft collision probability. These are the Foster (used by NASA), Patera, and Alfano formulations. We compare the numerical results over a very wide range of input collision parameters: miss distance x_e, cross-sectional radius r_A, and standard deviations $\sigma_{x'}$ and $\sigma_{z'}$. It is found that, over the present limits of the collision parameter ratios, the four formulations yield collision probabilities agreeing to at least 1% for the case of circular collision cross section. The NASA model is strictly applicable to the case of circular cross section, whereas the other three may be extended in principle to noncircular areas.

From the standpoint of the number of function evaluations, the Chan method is 1000 times faster than the Patera and the Alfano methods and 10,000 times faster than the NASA method. If collision probability is used in screening thousands of pairs of orbiting objects that are candidates for potential collision, this difference in computation speed is an important factor to be considered.

In addition, we also compare the collision probabilities for triangular and rectangular cross sections using the Chan, Patera, and Alfano formulations. Again, we find that the numerical results agree to two or three significant digits.

1.6 International Space Station Collision Probability

A little consideration reveals that it is possible to compute the collision probabilities for complex spacecraft structures having geometrical components such as plates, discs, spheres, ellipsoids, cylinders, cones, and parallelepipeds if self-shadowing is ignored. The approach is to obtain the collision cross section of each component and convert it to an equivalent circle having the same area and the same centroid. We shall refer to this as the method of equivalent cross section area (MECSA).

Presently, NASA models the International Space Station (ISS) as a sphere with radius of approximately 60 m. With the capability provided by MECSA, we may now model the ISS in greater detail. Instead of modeling every minute detail, we shall just model the solar panels and shields as flat rectangular plates, the main cross boom as a rectangular cylinder, and the other modules as circular cylinders. As before, we shall take the origin of the encounter system to be at the secondary object (an inactive or rogue spacecraft, or debris). Each component of the ISS is regarded as a primary object. The orientation and centroid of each of these components are known in the ISS body system. They are easily transformed to the encounter system by known orthogonal transformation matrices. It now makes sense why we conveniently chose the origin of the encounter system to be at the secondary.

In Chapter 6, using MECSA, we compute the collision probability between the ISS and an oncoming object for three case studies: head-on collision, broadside collision, and overhead collision. If the huge solar panels are tilted at 30 deg with respect to the relative velocity, we find that the collision probability for the broadside collision is one or two orders of magnitude smaller than that obtained for a spherical model.

1.7 Maneuvers to Mitigate Potential Collision Threats

Closely related to the problem of computing the collision probability between two orbiting objects is that of deciding, planning, and implementing maneuvers so that the two objects will have a reduced chance of colliding. Chapter 7 looks at these processes. The primary object is assumed to be an active controllable spacecraft. The secondary may not be controllable, as in the case of a rogue satellite or space debris. Suppose we are faced with a situation of high imminent collision probability or one that is greater than a specified acceptable threshold value. We wish to perform an orbit maneuver that is optimal in the sense that it requires the minimum velocity change to mitigate the potential collision threat and reduce the collision probability accordingly.

Because we have the zeroth-order expression for collision probability in terms of closest distance of approach, it is possible to invert this equation to yield the latter in terms of the former. Thus, if we specify a sufficiently low collision probability, we can compute the requisite separation at closest approach. From this, we can obtain intrack thrusting so as to decrease the collision probability to that specified acceptable threshold value. The analysis is simple and the maneuver is also simple to perform. Alternatively, we can also derive the equations for general thrusting with crosstrack and radial components. The analysis is complicated and the maneuver is difficult to perform.

The point to bear in mind is that, except for cases of immediate exigency, we should not base a maneuver only on myopic considerations of near-term reduction of collision probability, but we must also take into consideration subsequent scenarios. In whatever situation, we recommend performing a simple intrack thrusting.

1.8 Analytical Expressions for Long-Term Encounters

Certain pairs of geosynchronous satellites have been observed to have low relative velocity (so low it is measured in units of meters per second) so that the time they spend in the encounter region is appreciable. During this period, which can extend to well in excess of a day, neither the relative velocity can be considered constant in direction nor can the combined covariance error ellipsoid be treated as constant. In contrast to the existing short-term encounter models for which the assumption of constant relative velocity is valid, the long-term encounter model requires that a different approach be used for computing collision probability. Chapter 8 explores this approach.

In order to analyze the general case of long-term encounter, we must account for the covariance error ellipsoid changing in shape, size, and orientation. Thus, we can no longer consider the three Cartesian coordinates (x, y, z) representing the relative position uncertainty as random variables. Instead, we must choose three different random variables (x^*, y^*, z^*) as a common basis so that we can meaningfully compute the collision probability. This probability is determined by evaluating a three-dimensional integral over the appropriate region V^* corresponding to the values of the random variables. This is the crux of probability theory.

We can derive equations for the transformation of a sphere of radius r_A in the (x, y, z)-space to the (x^*, y^*, z^*)-space. We also need the relative motion of the primary in the physical space consistent with the laws of dynamics. As a first approximation, the relative motion is given by the well-known Clohessy-Wiltshire equations.

Next, we need to derive the equation of an envelope generated by a sphere of radius r_A moving in the three-dimensional space (x, y, z). The volume enclosed by this envelope is denoted by V. This volume is then mapped into the corresponding volume V^* in the (x^*, y^*, z^*)-space and is used for integration to obtain the collision probability. In general, the region of integration V^* is very complicated, being a tube with a curved axis and varying cross section. It may not be simply connected such as a torus, an elliptical doughnut, or even a pretzel. The collision probability in most cases must be determined by numerical integration.

1.9 Short-Term vs. Long-Term Spacecraft Encounters

Most encounters between spacecraft are short-term in duration, but it would be misleading to treat all of them as such. There are encounters that are of the long-term nature. By using equations for the two types of encounters, Chapter 9 examines cases to elucidate the differences between their results.

Three illustrative cases of long-term encounters are studied in detail in this chapter to compare the results of the correct long-term formulation with those obtained by inappropriately using the erroneous short-term formulation. For specific cases, one collision probability can be greater than the other or vice versa, depending on the range of collision parameters under consideration. Until the requisite computations are performed, it is difficult to make any a priori conclusions from an intuitive point of view.

In the first case study, we consider a family of in-plane concentric ellipses. Each ellipse describes the relative motion of a secondary spacecraft around the primary spacecraft. We find that the erroneous short-term encounter collision probability is larger than the correct value obtained for the long-term encounter when $(A/\sigma) < 0.5$ and smaller when $(A/\sigma) > 0.5$. The variable A is the semimajor axis of the ellipse, and σ is the standard deviation of an isotropic Gaussian pdf.

In the second case study, we consider a family of fixed-size in-plane ellipses along a straight line. Each ellipse describes the relative motion of a secondary spacecraft with reference to the primary spacecraft. We find that the erroneous short-term encounter collision probability is always larger than the correct value obtained for the long-term encounter.

In the third case study, we consider a family of variable-size in-plane ellipses with a common apsidal point. Each ellipse describes the relative motion of a secondary spacecraft with reference to the primary spacecraft. We find that the erroneous short-term encounter collision probability is larger than the correct value obtained for the long-term encounter when $(A/\sigma) < 0.6$ and smaller when $(A/\sigma) > 0.6$.

These three cases illustrate the fallacy of using the short-term encounter formulation to compute the collision probability when the relative motion does not meet the criteria of rectilinear motion.

1.10 Formation Flying

Chapter 10 describes onboard procedures to assess probable collisions between spacecraft flying in formation and between the constellation members and other orbiting objects. In addition to making these assessments, we would like to minimize collision risk by performing evasive maneuvers, and to design contingency plans for safe haven parking and subsequent system recovery. These efforts should draw on the analytical expressions derived previously for computing collision probability.

First and foremost in formation flying, we need to assess the collision between members of the constellation during flight. During normal times of operation, the satellite positions can be obtained by using the Global Positioning System (GPS) and intersatellite range measurements. During times of system failure resulting from malfunction of any spacecraft sensor, it is desirable to have other backup means of orbit determination for that particular space vehicle. The use of intersatellite ranging by the other functioning constellation members should provide this information.

For evasive maneuvers, we advocate using intrack thrusting for mitigating potential collision threats because it is much simpler to implement and execute operationally for spacecraft maneuvers. After the close encounter, it could be used again to regain the proper phasing of the spacecraft in its nominal trajectory. This is especially important if the primary spacecraft is one member of a large constellation moving in some prescribed configuration.

It is desirable to have contingency plans and to devise a safe haven configuration whereby we can set the system up for eventual recovery operations. Safe haven parking in times of emergencies should take advantage of the information provided by the estimation, guidance, and control systems from the running archives available on board.

For subsequent system recovery, we recommend that the onboard system make use of the information provided by the relative trajectory estimations of the satellites' state vectors during times of nominal operational conditions. In times of emergencies, the onboard system will take advantage of the information provided by the estimation, guidance, and control systems.

1.11 Maximum Probability of Collision

Chapter 11 considers the following problem: Suppose we are given the Gaussian pdf in the two-dimensional encounter plane. However, we are not given the standard deviations σ_x and σ_z and the cross-correlation coefficient ρ_{xz} or, equivalently, the two standard deviations $\sigma_{x'}$ and $\sigma_{z'}$ in the principal directions. All we know are the distance of closest approach x_e on the x-axis and the combined circular collision cross-sectional area of radius r_A with center $(x_e, 0)$. We would like to determine the maximum probability that can be obtained by assigning different values to $\sigma_{x'}$ and $\sigma_{z'}$ in the $(\sigma_{x'}, \sigma_{z'})$-plane. There are two approaches: the maximum likelihood method and the maximum probability method.

The first of these is an approximation. The principle of maximum likelihood can be used to obtain a quick estimate of the maximum probability when the pdf does not vary much over the combined circular collision cross-sectional area.

The second approach is an exact analysis of the problem as stated and depends only on the accuracy of the analytical expression for the collision probability previously derived. Thus, if we use the zeroth approximation of the collision probability, we expect approximately 1% accuracy for the present limits of the collision parameter ratios.

The numerical results obtained from the two approaches agree to two significant figures or approximately 1% over much of the range of applications interest. Thus, the simpler method of maximum likelihood can be treated as being of comparable accuracy for the present limits of the collision parameter ratios.

1.12 Close Encounters with Multiple Satellites

We are next concerned, in Chapter 12, with a detailed calculation of the probability of collision between the primary spacecraft and numerous secondary orbiting objects during a period of time. These secondary objects are grouped into different classes. The objects in each class have circular orbits and lie within small tolerances about a specific altitude and inclination; the value of the latter parameter can be different from that of the primary spacecraft. All the secondary objects in this class have ascending nodes uniformly distributed around the equator and have phase angles uniformly distributed within an orbital plane. Each class also has an associated number density of orbiting objects.

We can approximate the bivariate Gaussian pdf by an isotropic pdf because the angular direction with respect to the ellipse's principal axes is not known. Thus, the problem is again reduced to that involving the Rician integral. From this, we obtain the equation giving the number of collisions per orbit for each class of secondary objects in terms of its equatorial density, inclination, orbital period, relative velocity, and other collision parameters.

Next, we consider the geometry of the encounters in terms of the inclinations of the primary and secondary objects and the latitude of the encounter. We also categorize the collisions into approaching and trailing. For collisions of the primary spacecraft with all the secondary objects of a given class, computations reveal that the predominant contribution to the collision integral comes from encounters near the inclination latitude. (The inclination latitude is defined to be the highest or the lowest latitude reached during an orbit.) The contribution of the last 2 deg to the value of the approaching collision integral I_A is substantially larger in proportion to the rest of the contribution. The same statement holds for the trailing collision integral I_T. For most of these cases, this contribution is approximately 25% or more. This larger contribution means that there is a greater probability of collision near the inclination latitude.

1.13 Instantaneous Probability of Collision

Up till this point, we have considered the probability of collision between two orbiting objects during a time interval, whether it is the case of short-term or long-term encounter. The probability integral was performed over a volume of the random variable space correspondingly swept out by the motion of a sphere of combined radius $r_A = (r_p + r_s)$ in the physical three-dimensional space. In the case of short-term encounters, the volume of integration was an infinitely long circular cylinder. In the case of long-term encounters, the volume was more complicated, being that contained in the enveloping surface traced out by a time-varying ellipsoid. We used a torus and an elliptical doughnut as simple illustrative examples.

Chapter 13 addresses the idea that, in certain other applications, we may be required to determine the probability of collision at a specific instant of time. In this case, the volume of integration is a sphere of radius R that may be the combined radius r_A or some other parameter such as an effective "event radius" r_E. This event may be the sudden misbehavior of a rogue spacecraft or the explosion of a nearby orbiting object producing many fragments. Inside a sphere of radius r_E, an object can be considered as being impacted by the rogue spacecraft or exploding fragments within a very short time interval.

A little consideration will reveal that the analysis given here is not restricted just to the probability of collision between two orbiting objects. It is equally applicable to other three-dimensional phenomena, such as the explosion of a fragmentation bomb near Earth's surface.

To compute the probability of collision, we can generalize the previous approach by using the transformation of a Gaussian pdf to a Rician pdf and introducing the concept of equivalent volume. It is found that the accuracy is within 1% throughout a wide range of collision parameters. On the basis of the number of function evaluations, which is an indication of computing time, this method is found to be approximately 65,000 times faster than the high-precision brute-force method of numerically evaluating the three-dimensional Gaussian integral.

1.14 Spherical Error Probability Computation

Suppose we are given a pdf and a spherical volume of a certain radius. We can determine the collision probability by evaluating a three-dimensional integral either analytically or numerically as we have just done. This is known as the direct problem. However, we may be given the inverse problem: We need to determine the radius of the spherical region when the collision probability is specified. This problem is known as spherical error probability (SEP). Chapter 14 focuses on computing it.

Six parameters are input to the problem: three standard deviations of the Gaussian pdf, and the three components of the displacement vector of the center of the sphere of consideration. The most obvious approach to determining SEP is to perform many time-consuming numerical evaluations of three-dimensional integrals and then interpolate to obtain the desired result.

The approach here is radically different from that just described, in that we use concepts from statistics to invert an expression involving a particular type of integral. It is based on first formulating the problem in terms of the cumulative noncentral chi-square distribution; then appropriately choosing the degree n' of a central chi-square distribution to approximate the noncentral one; and last, in a second transformation, transforming the central distribution to a Gaussian distribution. The degree n' is chosen so that the first three moments are satisfied. The transformation from central chi-square to Gaussian distribution is accomplished using an expression derived by Wilson and Hilferty. The radius of the sphere is obtained through the use of the inverse error function.

Numerical results are presented for more than 200 benchmark cases. Comparison with several cases of known exact analytical solutions indicates that this method is generally accurate to approximately 1% for probabilities greater than 0.5 and approximately 10% for probabilities between 0.1 and 0.5. This level of accuracy meets the requirements of most applications.

It is recommended that this analytical approximation be used when it is necessary to evaluate very high-dimensional integrals of this nature arising in statistical analysis and signal detection.

1.15 References

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Spacecraft Collision Probability Analysis: Theory and Problems Dr. Matt Hejduk 3:30 p.m. â€“ 4:30 p.m. Thursday, September 26th, 2013 GL51, Marrs McLean Science Building The Department of Statistical Science welcomes Dr. Matt Hejduk to campus to present â€œSpacecraft Collision Probability Analysis: Theory and Problems.â€Â Collision Avoidance and Risk Assessment (CARA) project at NASA Goddard Space Click here for full PDF collision probability non-zero cross correlation filter orbti determination ellipsoid Publication Date: February 2004. What We Offer. Satellite Design and Operations. Space Support. Intelligence Analysis. Aircraft Design and Operations. Missile Systems.