Geometry and Art, What a Concept!

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WHAT WILL THEY SEE?

"Now class, visualize a cube . . ." Some pupils will immediately think of a piece of ice; others can see a three-dimensional cubed shape in their mind's eye; yet other children will still see a flat square. An ordinary cube is made up of six squares; however, so many students fail to establish an articulate difference between a square and a cube. Geometric forms are numerous and perhaps confusing to a young mind that hasn't yet formulated a real sense of perceiving dimension, vocabulary, or comprehension of forms.

Connecting Different Cognitive Skills Can Connect the Dots . . .

My interest in integrating and emphasizing geometry is twofold. Geometric shapes are some of the strongest forms used in modern art. Geometry has a strong history that, throughout the centuries, has been involved, integrated, and eventually indispensable in art and architecture. Artists have studied geometry in order to draw angles, proportion, and perspective, in order to illustrate or emote the illusion of realism. Geometric forms, currently, are explored and seen everyday in our surroundings. Geometry is an important subject, one which plays a strong part as a tool in art and needs to be emphasized at some point in a person's education. Students can benefit from the awareness that art and math share a significant part together.

Surely, a secondary reason for weaving geometry into an art unit is to strengthen and complement both subject matter skills. The practice and repetition of maneuvering shapes and forms and then relating them to math vocabulary will enhance knowledge for those who already have a strong sense for forms, angles, and dimension. For the unexposed student, accomplishing and replicating geometric forms will enable the time for a good deal of exploration to grasp understanding. This unit provides a progression of lessons, moving from a study of two dimensions to an investigation of three dimensions. The lessons provide a chance to explore art history as it relates to the use of geometry in addition to a fundamental review of sixth- and seventh-grade math objectives in geometry. I will also suggest preliminary activities with materials and manipulative ideas to enhance productivity in later lessons.

Some Really Like Art

Each semester, I see a variety of students. Some have an interest in art, some are really talented, some are gifted academically, and then there are the students who are just not interested in art at all! My unending quest is to find ways to teach and reach all types of

students at the same time with the subject matter presented. For me, this means a constant search for ways to interest the learner through different approaches.

I've searched for tactics that introduce my subject matter in a way that is motivating/captivating and appealing. I desire to peak students' interest and have them reach an acceptable level of accomplishment; to that end, choosing geometry together with art will assist me greatly in solving and presenting a unit that meets all my students' needs and becomes a regularly used component in my classroom.

This combination of geometry and art will not only target special, talented pupils, but will also hopefully generate curiosity and action from the less interested students. The plan is to provide energetic, interactive, and constructive games that can increase students' comfort level as well as their math and drawing skills.

A Way for All to Function

There are several components in my unit that will step up my pupils' achievements. There will be a time to explore and comprehend, an opportunity to research, a chance for skill practice, and time to create their own project to show proficiency.

This unit has the student complete three steps that are the same within each section of the art/geometry lessons. First, an introductory slide show, video, or PowerPoint presentation will be shown to the class for each different sections/lessons involved. After objectives are explained, the student will have a chance to research through books and the Internet as well as practice math problems or art skills. They will answer, write, or work out problems to hand in within the lesson.

After a sufficient amount of practice/research, they will be ready to do the second part of the lesson, which is to create the actual project intended.

The third component is available at several of the prepared centers that are available around the room for further exploration. These versatile centers will be available for the quick-to-finish, gifted pupils, or for the student who really needs further practice and comprehension.

These lessons were prepared to try to reach specific objectives. They will provide both strategies and opportunities for non-exposed students, gifted and talented kids, and regular children! These objectives are as follows: for all students to gain a balance in their learning, in order to reach their potential, think independently, interpret data, practice the act of predicting, work cooperatively when in groups, share and contrast ideas/solutions, apply art knowledge, use critique methods necessary for effective non-verbal and verbal communication, and use sensory learning methods in order to increase understanding of the relationship that math has to fine arts study.

What Will They Know?

Through the arrangement of projects, I am looking for my students to understand and integrate geometry and spatial reasoning as well as be able to acquire better drawing skills, analyze works in progress, demonstrate an understanding of how geometry is used in masterpieces, and make informed judgments of their or others' artwork. The projects planned are age appropriate, interesting, and challenging. Hopefully, the student will be engrossed and will look forward to each step they take in learning and creating each project:

Projects help the learner to interact with knowledge at a level higher than simple recall. They also help develop concepts more fully and enable students to construct their own understanding. Information learned in context with an emotional hook will be remembered longer if it is also a meaningful experience. Projects emphasize process as well a product and integrate many concepts, facts, and skills (Gregory 120).

What Can Be Gained?

So much information can be uncovered, from necessary vocabulary in art and math, to historical knowledge of artists and mathematicians, to basic formulas and the ability to identify them in artwork, to obtaining a better understanding of geometry through the manipulation of art tools.

Furthermore, after the unit is employed, my intent is to have the student develop a more critical eye and be more secure at what he thinks about mathematics, art and its relationship. The world is full of geometry, from natural phenomena to manmade forms. I hope to see that my students increase their involvement in art class through the *new* use of geometry and for this to have a positive effect on students who have been uneasy about mathematics as well.

It is my intention to challenge my students as well as myself to identify when geometry is being used in artwork. Before starting a work of art, artists set up problems for themselves as a way to begin. These may be parameters of sorts, from color choices, to shapes, to types of forms, to a theory of how to implement, etc. For the artist, the planning stage is always a brainteaser that they are interested in solving. They have numerous variables to use or choose from in order to solve and finish their painting, drawing, or sculpture. By seeing and considering the artwork of others, my students will be identifying, synthesizing, and evaluating samples that use geometry.

After some practice in recognizing the artist's attempt to solve his/her problem, I believe that my students will find it easier to recognize which type of parameters the artists set up for themselves. Then, students can check to see if the problem has been solved adequately by a given artist by analyzing the proportions, symmetry, and use of geometry. Once the artwork's challenge is defined and solved, a student can set up

similar projects to solve mathematically as well practice in the techniques of drawing them.

I can envision the students becoming comfortable with the process and reviewing some of the great murals and paintings from the Renaissance, then moving towards more challenging works like those of M.C. Escher, Victor Vasarely, contemporary artists like Dick Termes, artists/mathematicians like Mike Field, or sculptors/mathematicians like George W. Hart or Alan Linder.

There will be time to explore one-, two-, and even multiple-point perspective artwork, and then move on to three-dimensional artworks that include geometry as a focus. Students will do activities that will give them a chance to experiment with creating the illusion of three dimensions in a 2-dimensional space. More interactive activities would include Internet research of the artists, rendering dimensions, doing tessellations, or working a geometric project within groups or individually, using computer software.

How Will They See It?

There is value in using two-dimensional and three-dimensional, "hands-on," manipulative, interactive activities and creative centers in order to link vocabulary or set understanding of geometrical shapes before you can anticipate a development from all the compiling, connecting, and synthesizing the student will do with the new information introduced in the unit. Short activities like the ones prepared here will help the student form new relationships before they interact in the more sophisticated tasks and challenges presented in the actual lessons.

We as teachers try to cover, prepare, and use all that we can that has to do with the inclusion of "multiple intelligence activities" in our lessons. This unit addresses the challenge. In the article "Developing Multiple Intelligences in Young Learners," Connie Hine explains the benefits of providing and exposing children to meaningful learning:

We can foster meaningful learning experiences by using multiple teaching tools and strategies that make learning more efficient and by building positive, supportive, relationships with children. Through environments that offer a variety of stimulating, hands-on materials that children individually select and by creating learning centers that provide natural opportunities to move, be active, and fully engaged in either solo or small group experiences, we better serve and meet the needs of more children (Hine 5).

In providing a collection of worthwhile manipulatives, I feel that the following requisites are necessary. The selection of items that one obtains for the student to operate should have several levels of complexity. Mini-lessons should be attached to these learning centers in order to make sure that the student is progressing satisfactorily. The manipulative should be age-appropriate or age-attractive. These materials need not necessarily be commercially made. I do not reject the teacher-friendly, ready-made math

manipulative. I would just add some non-traditional type of materials like drinking straws, plastic utensils, found objects, playing cards, dice, plastic string, aluminum paper, beads, and any other found or affordable items that lend interest to the learning center.

As an instructor of art, I find that it is very beneficial to have material experimentation prior to any art project. Learning centers and investigation of math and art manipulatives will be imperative before students can learn and do the various projects successfully. The combination of several types of materials will allow for the development of comprehension for some and the ability to use imaginative/higher-level skills for others at these experimental centers:

Next, we can alter our instruction to provide many different types of strategies, activities, and experiences – something for everyone. Different group configurations, opportunities to move and interact with learning materials (for example, learning centers), structured and unstructured activities, verbal explanations and visual resources are all examples of different approaches (Shalaway 60).

A Place to Experiment

Designate in your classroom a place that is separate from regular class activity. Place a center or two. It can be the ideal starting or ending place (that, you will choose) for certain students to spend for reinforcement and for others to work on more projects because they are finished before the class; alternatively, it can be assigned as a special time for all students as a rewarding alternative.

In preparing the area, it is important that you have decided what the main reason for the center is as well as how you utilize the center; then, prepare the area with mini-lessons so that the activity center/s can be successful for your needs. In other words, adjust it/them for your specific pupils' needs.

Mini-Lesson Center Ideas

Exploring Geometric Shapes, or Lesson in a Box!

1. Seal a box and cover it. Make sure the box has an opening for just one hand. Inside, place many written geometric shape names on small pieces of paper. If the center is geared for a beginner's level, the shapes should reflect what the child needs to understand. If the center is geared for gifted learners, the shape choices should be challenging (from creating a simple polyhedron to maybe giving a challenging specific like a rhombicuboctahedron!).

   All centers should have a 3-D geometric building type tool (manipulative such as Zomer – found in Math manipulative teacher catalogs), a reference book (a book

about geometric shapes, a school math book, or various other math resources), notebook paper, and a copy of the written mini lesson to be completed.

2. Have the designated student/s pick one from the box. (The teacher keeps the box!)

3. The student/s will go to the center to work out the problem. Once it is completed, they need to write down and give the definition of the shape and give the shape created and paper to the teacher for extra points or another form of acknowledgement.

4. Opportunities to create quizzes from what ever is being learned/achieved by the pupil are endless. Here are some other ideas for the Lesson in a Box activity…
   - Change what is in the box. Put the definitions only in the box and have the student build the geometric form from the definition. Tell the teacher what the name of the specific geometric form is from its description. The materials in the center are the same!
   - How about a mini-lesson that requires students to create a geometric form made from bendable / and non-bendable drinking straws…
   - Use playing cards as a manipulative to create tiling
   - How about forming geometric forms from rectangular shapes (gluing is an option)

Building Art Skills Center

Here is an idea for building drawing skills. Get some heavy cardboard in order to be able to glue some items on this board to serve as a base. Found objects like wooden blocks, geometric forms, empty plastic bottles, cardboard cylinders, plastic action figures…. Glue these items in interesting ways. Set them on top of each other, stacked together, in a pattern…be creative, your choice. Make sure that these items are glued on to the heavy-duty cardboard for easy handling. Make several types of these "found objects" manipulative.

These art skills centers should have paper, pencils, pens, rulers, erasers, mini-lessons, reference art books, and or watercolors at hand.

Mini-lessons can include from achieving rudimentary drawing of angles, to using linear perspective, to adding shadows or using different art materials such as pen and ink or watercolors. (See drawings from M.C. Escher for reference) Again, depending on what is needed lower-level or higher- level learning.

1. Give the student a board at a time for drawing. Have drawing paper, erasers, rulers, pencils and a copy of the mini-lesson at the center. Let them try to draw for themselves and a time to practice.

2. As the student gets more time, let them continue trying to draw the more difficult boards. This will enhance their abilities as well as their confidence.

Exploring the Essentials . . .

There is some information that an instructor has to review in order to prepare to teach any part of this unit. It has to do with understanding how both kinds of subject matter mesh together. Geometry deals in two dimensions as well as three.

Throughout the centuries, the use of geometry was one of the artist's precision vehicles in creating illusions. In order for the artists to create in the Renaissance, they had to learn geometry as part of becoming an apprentice in the art and science guild. Euclidian geometry, linear perspective and the use of the Golden Mean measurement were some of the classic formulas used to understand depth, volume, mass and how to manipulate these to mimic nature when creating their works of art. In the listed reference books by Alison Cole on the Renaissance and on perspective she relates about these formulas and the artist that employed them:

In Regards to Artist Piero Della Francesca

It is thought that Piero used Euclid's ancient formula for constructing a 15-sided polygon as the mathematical basis for the Baptism of Christ (Cole, *Renaissance* 32).

Piero Della Francesca (c.l4l5/20-1492), inspired by his belief in the perfect geometry underlying God's creation, devoted much of this life to the study of geometric solids and the mystical properties of numbers (l8).

In Relating to Perspective

Linear Perspective – a mathematical system for representing three-dimensional space on a flat surface – was devised in Florence in the early l5th century (Cole, *Perspective* 28).

The artists that I've chosen to present to my students are not just only very well known, but have easy, accessible examples in art history books, not to forget, numerous samples of works found on internet. Ideas on where to find good samples of the artist's suggested works are listed in the lesson plans.

Gathering up the Proof

In regards towards finding good samples to show my students, here are some pictorial references that I used. The Renaissance artists who involved Euclidean geometry in their compositions were Leonardo Da Vinci, Piero Della Francesca and Albrecht Dürer.

Linear perspective illustrations can be found in most works of Masaccio, Leon Battista Alberti, Bramantino and Filippo Brunelleschi. Artists using the *Golden Mean* or called "*The Divine Measurement*" (known today as the Golden section, a way to measure proportions) can be found by works of Piero Della Francesca, Leon Battista Alberti or

Paolo Ucello. In progressing towards 2 dimensional modern works, I would find art from M.C. Escher, Victor Vasarely, and present-day Dick Termes and Mike Field. These are my suggested and preferred artists for collecting exemplar samples to use.

In moving towards exploring of 3-dimensional artwork, Early Greek or Roman art can be introduced along with Renaissance art and sculpture pieces from Michelangelo Buonarroti, and the architecture drawings or buildings of Filippo Brunelleschi. Geometric forms as sculpture were not really explored by artist until the l900s. What was used was geometric measurement in order to create sculpture.

A similar progression towards involving the students in discovering 3-dimensionality with geometric forms will proceed from the last century to current day artists. Artworks by artists such as Isamu Noguchi, Alan Linder, Alexander Calder or George W. Hart can be researched by the student. From student's Internet research, about sculptor's works to creating their own geometric sculptures . . . students will be exploring various ways that geometry and art perform together.

Some Initial Thoughts

Today, it is still difficult to separate perspective drawings from geometric forms and angles. Geometry is one of the key tools that artists still utilize in order to create the illusion of three dimensions on a two-dimensional medium – that is on a wall, drawing paper or canvas. Proportion, patterning, and symmetry, as geometrical tools, are not far behind needed, in creating successful three-dimensional illusions. The use of proportion and symmetry along with other geometric operations, are in fact, the very tools we use continuously in order to understand what we see visually. We as humans do this unconsciously when we are observing artwork or architecture. We may not know why a painting or a building looks awkward or different to us, but what we really are observing is the proportions, angles and symmetry of the structure or form.

The study of geometry has many options: the study of proportion, symmetry, planes, the study or recognition of shapes, movement with shapes, looking for harmony, understanding various forms as two-dimensional or three, spatial relationship, patterning and so much more. Students in any art curriculum deal with the same concepts: proportion, symmetry, planes in 2 or 3 dimensions, movement of shapes or forms, patterns, and spatial relationships. Art and geometry really have very much in common. Looking back to the Renaissance artists, we can see that they were lifelong students of their craft. They believed that you needed to seek, study, and know just about everything that you could in order to perfect oneself. That is why today, we call this era the "Rebirth" of knowledge. Art and Math, especially geometry, were definitely intertwined. In an artist's apprenticeship, one studied the early Greek and Roman classics, such as Vitruvius, the architect, Euclid, the mathematician, and some of their contemporaries like Luca Pacioli.

Did Pacioli then look entirely towards the past? Certainly not. He was also a man of his time. His passion for mathematics was that of a humanist. He admires its sure progression towards the truth, so satisfying to the human spirit. Hence his devotion to Euclid, whom he always calls "our philosopher" and to whom he constantly refers. (Bouleau and Villon 32)

There are examples of Leonardo Da Vinci's journals where it shows that he drew and studied reflection, patterns, perspective, and measured the movement of objects. Along with his observations and drawings, he was acquainted with Luca Pacioli's work. This mathematician wrote about geometry and the golden rectangle rules of proportion. Pacioli wrote math papers about Euclid's formulas. Artists like Piero Della Francesca and Leonardo Da Vinci put a lot of these teachings into their artwork.

Putting geometry formulas into their art was not only a way to study how one could make illusions look real, but also to try to prove through drawings and observations that Euclidean geometry was a true rule in ways to see the world through patterning, proportions, scaling and the use of tiling, etc. Later on, in the late l800s and early l900s, artists by the name of M. C. Escher and Piet Mondrian took this same way of observing reflections, patterns, movement and the use of geometry's formula for the Golden Rectangle yet to another level.

The inclusion of some history of geometry must at some point presented and/or included with the introduction of the employment of geometry in artwork. This will be imperative in order to familiarize the student with a sense of where math and art merge. Through slides, books, films, internet research, and a trip to the Museum of Fine arts, I plan to expand their knowledge and confidence in deciphering and acknowledging where geometry is used, linear perspective, golden ratio, symmetry, etc., and be able to analyze the success of the parameters set by the artist. I think this will excite and challenge my students. Once they start creating their own drawings with some of the same strategies, I believe that it will as well, connect them with the world of math. This unit should provide a catalyst for those afraid of math and for the more academic minded child, a learning process towards having fun with the act of drawing.

A Starting Point: Uncovering What Our Eyes Really See

Introduction to Perspective

- The illusion of three dimensions in a two-dimensional setting.
- Slides/posters recommendations to use as samples or to create a power point presentation.
- Illustrations recommended as examples to use:
  - Bramantino, *Adoration of the Kings* (1498)
  - Paolo Ucello, *The Hunt in the Forest* (1460)
  - Piero Della Francesca, *The Baptism of Christ* (1450)

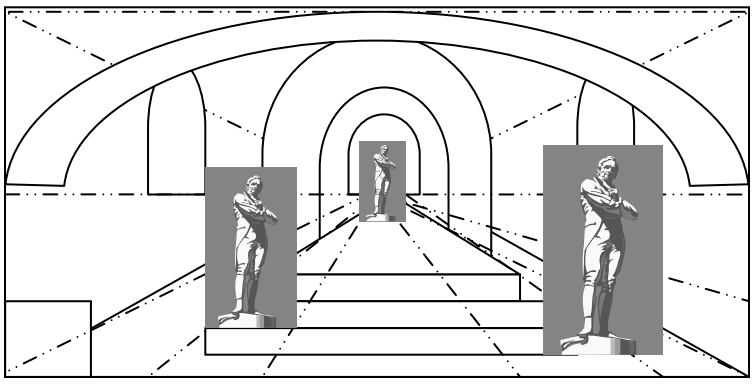
- Piero Della Francesca, *The Flagellation* (1460)
- Masaccio, *The Trinity* (1427)
- Raphael, *The School of Athens* (1509-10)
- Leon Battista Alberti, interior architecture of S. Andrea in Mantua, Italy (1470)
- Filippo Brunelleschi, interior architecture of S. Lorenzo in Florence, Italy (1417)
- Leonardo Da Vinci, *The Last Supper* (1497)

From these slides one can break them down like the one below. The teacher can break the paintings down to show the linear perspective and show the geometry used. The drawing below is a linear dissection taken from the "The School of Athens" by Raphael, but it does not include the people that are in the painting. It shows the central "vanishing point" or vertex.

What is Perspective?

Perspective is a system for representing three-dimensional space on a flat surface or plane. A pictorial solution used by Ancient Greek, Roman and was fully developed in the l5th century by pioneering experiments of Filippo Brunelleschi and Leon Battista Alberti. These two Florentine architects have been credited with its invention or rediscovery. The architect Filippo Brunelleschi lived from 1377-1446 and the architect/sculptor Leon Battista Alberti lived from 1404 to l472. The terminology for this type of perspective drawing is now called linear perspective.

Alberti compared the picture surface to ". . . as an open window through which the subject painted is to be seen" (Cole, *Perspective* 29.) Objects can be plotted by scaling the image by the use of a vanishing point. In geometry this is often called a vertex.



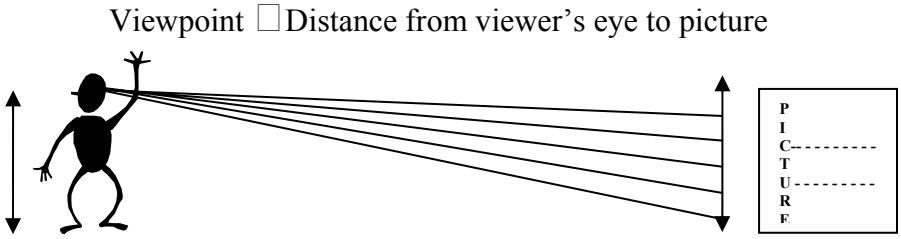
According to Webster's New World Dictionary, these words are defined as follows:

- Linear perspective: a mathematical system for representing 3-dimensional space on a flat surface (1062).
- Vertex: a point of intersection between two sides of an angle, a corner point of a geometric figure bound by lines, planes or lines and planes (1579).

- Vanishing point: the point where parallel lines receding from the observer seem to come together (1570).

Step 1: Alberti's System For Creating Linear Perspective

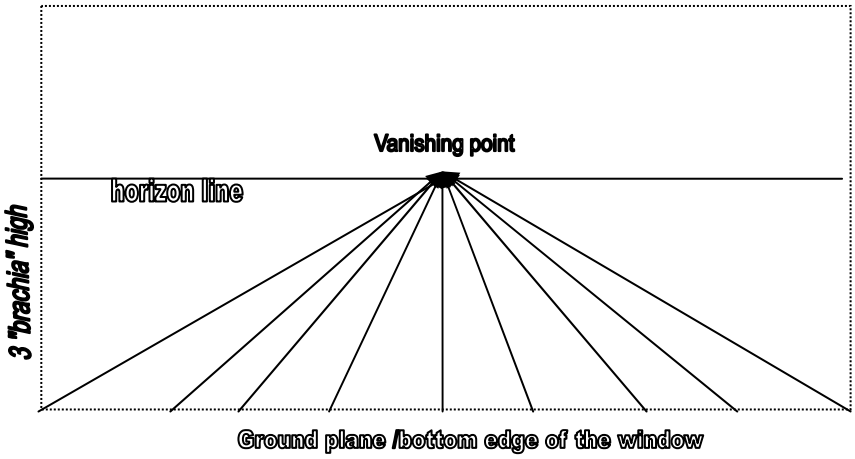
Alberti based his system on the height of a human figure. He set the average height at three *braccia*. A *braccio* was a Renaissance unit of measurement.



Leon Battista Alberti felt that in constructing a picture's perspective you would have to imagine the picture's surface as a plane cutting through a pyramid of visual rays.

Step 2

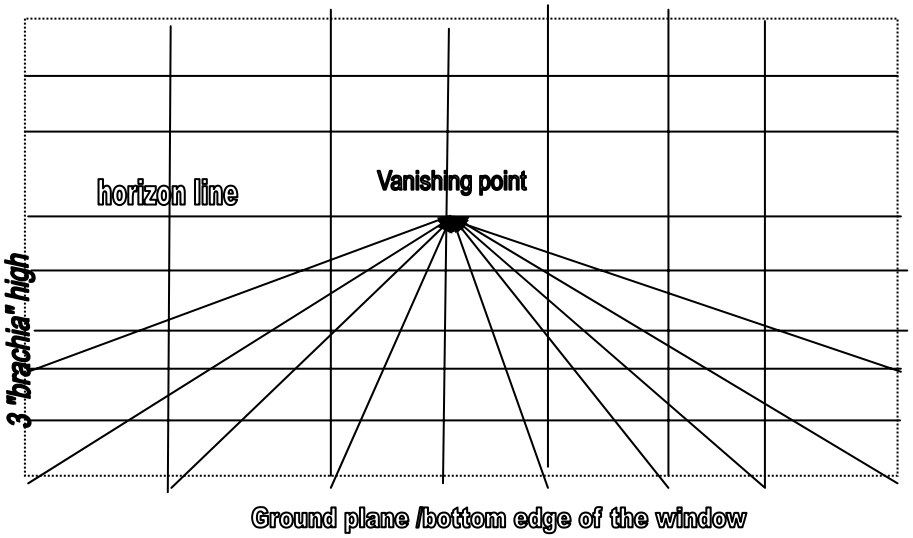
Having drawn a rectangular picture area, he divided the ground line into scaled *braccia*. Next, he would place the central vanishing point by drawing a vertical line, about 3 *braccia* high from the center of the ground line.



The front view of a picture is shown above. Imagine this to be an open window where one can see things that are in close proximity closer to the ground plane/bottom edge of window and see farther away as we get higher on the picture plane (closer to the vanishing point).

Step 3: The Checkerboard Grid

Having created a flat checkerboard type or tile pavement by drawing horizontal lines going all the way up to the horizon line . . . (representing the ground's plane levels called "transversals"), Alberti would place objects onto this grid using his one *braccio* to a tile scale. Depending on the height and width of the object, he would scale it by counting tiles across the plane, up the foreshortend checkered grid/plane. This is a very simplified explanation to a more precise measurement system that Alberti infact, implemented in his drawing.



LESSON PLANS

Lesson One – Introduction to Perspective:
Illusion of Three Dimensions in a Two-Dimensional Setting

Introduction and Activities

A. **A history review** of the Renaissance artist and Mathematicians. (Introduction/Western Style of Composition.) **Discover** geometry in drawing 3-D illusions (film, slides, posters).
B. **Explore and discuss**: (w/overhead projector to find all keys to creating a perspective drawing or painting) the use of lines, angles and all vocabulary words can be reviewed about linear perspective through dissecting a Renaissance painting. A suggested sample to use: Raphael, 1509-10, *The School of Athens*
C. **Vocabulary and Questions Find Sheet :** symmetry, parallel lines, congruent, balance, plane, shape, perpendicular lines, vertex, vanishing point, angle, ray, intersecting lines, line of symmetry and proportion
D. **Practice**: Student drawing of a 1 point perspective, vanishing point with parallel lines. Drawings will include different sizes of cubes and rectangles using proportions

Lesson One Activity Sheet

Name_____

Vocabulary words can be found in the dictionary, as well as in math and art reference books in the classroom.

Date _____

General Objectives and Time Allotment

The student will address for themselves . . . learning and clarifying new vocabulary, acquiring an awareness of geometry, art tools, and 2 dimensional perspective drawing skills. They will have an opportunity to recognize and describe when geometry is being used in 2-dimensional artwork, gaining control using measuring tools, rulers, t-square, pencils, and triangles. The student will do some research on their own . . . through books, CD-Rom, and Internet. This lesson is designed to be a 1-week session.

Concepts and Vocabulary to Research

Some vocabulary words will be repeated in future lessons for reinforcement of understanding.

symmetry	parallel lines,	congruent	balance
shape	perpendicular lines	vertex	angle
ray	intersecting lines	symmetry	proportion
vanishing point	plane		

Helpful Websites

- <http://www.vitruvio.ch/arc/masters/vitruvius.php>.
- <http://math.truman.edu/~thammond/history/Vitruvius.html>.
- <http://educ.queensu.ca/~fmc/october2001/GoldenArt.htm>.

Words to Know

Look up these vocabulary words in your dictionary, art books or math reference books or on the Internet (for example, <http://www.mathleague.com/help/geometry/polygons.htm>).

plane-

vertex-

vanishing point-

perspective-

grid-

parallel-

linear perspective-

angle-

ray-

intersecting lines-

symmetry-

proportion-

illusion-

dimension-

horizon line-

vertical-

1. Who is Euclid?
2. What are some of the ideas is he famous for?
3. Who are Luca Pacioli and Vitruvius?
4. What is the "Golden Mean?"
5. What do artist and mathematicians of the l5th century have in common?

Answers for Lesson One Activity Sheet

Vocabulary Words
The words are found in basic reference books such as dictionaries, art and math books or website reference.

Research on the Internet and Reference Books

and

Name_____

Date _____

In your answers please write where you found the information: What book? What website?

plane	angle
ray	intersecting lines
proportion	illusion
horizon line	vertical
symmetry	linear perspective
dimension	perspective
vertex	vanishing point
grid	parallel

Questions and Answers

These are basic answers that can found to sufficiently relay the answers. Of course, depending on the student and their ability to research will the answer vary from a simple sentence, to a completed summary.

1. *Who was Euclid?*
   Euclid was a very famous mathematician of ancient Greece. He basically dealt with geometry. He created many books on his theories. Renaissance mathematicians, artist, and architects became aware of his ideas and studied his theories. The artist tested these ideas in their paintings, sculptures and architecture of the l5th century.

2. *What are some of the ideas is Euclid famous for?*
   Euclid was famous for creating the "postulates" or theories. The definitions of Euclid's first book, include those of points, lines, planes, angles, circles, triangles, quadrilaterals, and parallel lines. The first five postulates in his first book are the most remembered. They may be translated into the following:
   - Two points determine a straight line.
   - A line segment extended infinitely in both directions produces a straight line.
   - A circle is determined by a center and distance.
   - All right angles are equal to one another.
   - If a straight line falling any two straight lines forms interior angles on the same side less than 180 degrees, the two straight lines, if produced indefinitely, will meet on that side.

3. *Who were Luca Pacioli and Vitruvius?*
   Luca Pacioli was a mathematician that worked with Leonardo Da Vinci. He worked closely and wrote about Vitruvius, a Roman architect, and Euclid a mathematician. He was very involved with geometric relationships and theories. The ideas that he was most involved with were the classical formulas of proportion, mainly about the square and circle being an extension of the human body.

   Vitruvius was a famous Roman architect that wrote books on how to create buildings as extensions to oneself. He studied the Classics, also. He wrote about how every part of building or home was affected by and should accommodate the person living in it. Every part of each room affected the other. It was the first time one considered that function and place did matter. It spun a lot of ideas about spatial relationships.

4. *What is the "Golden Ratio?"*
   The "Golden Ratio" was a geometric system that some believe to be perfect. Still used today, the formula is found in nature or natural objects such as shells. For mathematicians of the Greek and Roman era, it was a unique and very important mathematical discovery. The Golden rectangle is that the ratio of its length to the width equals to approximately l.6l8. Golden Ratio = length = 1.6 width

5. *What do artist and mathematicians of the l5th century have in common?*
   The 15th century Renaissance Mathematicians and artist were using geometric systems to solve their answers or solutions. A very simple clear example/answer is found in "Math and Art: The Golden Rectangle" at: <http://educ.queensu.ca/~fmc/october200l/GoldenArt.htm>.

Lesson Two – Drawing Illusions is Good Geometry

General Objectives and Time Allotment
The student will address learning deficiencies; acquire drawing skills and use of measuring tools, rulers, t-square, pencil, triangles. The student will do some research on their own… through books, cd-rom, and internet. This Lesson is designed to be a 1 month session.

Concepts and Vocabulary to Research
Some vocabulary words here are repeated for reinforcement of understanding.

Renaissance Artists
Leonardo Da Vinci, Raphael, Paolo Ucello, Piero Della Francesca, Masaccio, Filippo Brunelleschi, Bramantino, and Leon Battista Alberti.

Terms

Vanishing Point	Vertex	segment	perpendicular lines
Parallel lines	intersecting lines	right angle	acute
Obtuse	degree	line	symmetry
Scale	grid	value	chiaroscuro
Perspective	pictorial space	color tones	variety of size and shape

Project Objectives
Provide opportunity to apply skills, concentrate on proficiencies, stressing intrinsic motivation. After the introductory work, information disseminated, and some experimentation… the student will show comprehension from composing, translate ideas, interpret and demonstrate by their own example, a 1 point and 2 point perspective drawing.

Part One: 1- Point Perspective Project
The students will create a 1 point perspective drawing of their own; showing proficiency in using drawing and measuring tools, understanding pictorial space through the use of geometric shapes, variety of lines and tones. The composition will show a vanishing point and a grid formation. The composition will have parallel, perpendicular, intersecting, obtuse, acute, right angle, segmented, and a variety of other lines. Application of pencil and the use of values will demonstrate *beginning skills* in developing 3 dimensional illusions through shading (chiaroscuro).

Materials

- a variety of measuring tools, rulers, t-square, pencil, triangles
- drawing paper, eraser and a variety of pencils: h, 3h, hb/2b, 4b, 6b, and ebony
- It would be helpful to have a still-life of cubes, rectangles, and spheres (wooden math manipulative kit). If not available you can use a variety of size boxes, plus some beach balls, soccer and/or basketball

Centers Objectives

Encourage student to determine what to work on and accepts them as experts about their needs. Work completed at centers will earn extra credit, (an incentive for both students that need to have more comprehension and skills.)

Students will be able to work on different centers to have an opportunity to practice, discover, and become more proficient. Geometry centers will have additional projects and books for research. Students will also have the chance to go beyond their composition project and create a 2 point perspective drawing for extra credit, time permitting.

For Re-enforcement

<u>Research from Reference Books</u>

Have the student do some of the chapters in the math or art book that provide information for practice.

<u>Research from Internet</u>

Have the student go on the Internet to do some research on any of the subject matters introduced, depending of the student's need. Look up: perspective, 2 dimensions, math terminology, mathematicians or artists. But have it be something ***specific*** to write about.

For Enrichment

- Have the student look up an artist, such as Dick Termes or M.C. Escher, through the Internet.
- Have the student create and compose another drawing with a 2-point perspective.
- Suggested web sites for this part of the lesson:
  - <http://www.art.com>
  - <http://www.louvre.fr/louvrea.htm>
  - <http://www.moma.org/collection/index.html>
  - <http://www.nga.gov/kids/kids.htm>
  - <http://www.nga.gov/home.htm>
  - <http://www.nga.gov/kids/zone/zone.htm>
  - <http://www.etropolis.com/escher/>
  - <http://www.termespheres.com/termes.html>
  - <http://www.mathacademy.com/pr/minitext/escher/index.asp>
  - <http://www.artcyclopedia.com/artists/escher_mc.html>

Now, For 2-dimensional / 3-D Illusions in Modern times
The emphasis is in introduction to Modern works, Color theory (complimentary), and how geometry is still being used (in its most elemental manner) in the creation of modern works. Show students a power point or slide show of the following recommended works: These samples can be gathered on the listed websites.

M.C. Escher, 1953, *Relativity*,
M. C. Escher, 1955, *Convex and Concave*
M. C. Escher, 1937, *Still Life with a Street*
Salvador Dali, *Sacrament of the Last Supper*
Salvador Dali, *Apparition of the Face of Aphrodite*
Victor Vasarely, 1989, *Tukor Ter Ur*
Victor Vasarely, 1968, *Axo-GJ*
Mike Field, 2000, *Studies in Blues and Greens II*
Mike Field, 2000, *Earth lines*
Dick Termes, 2003, *Perspective from Geometry*
Dick Termes, 2003, *To build a Hole*
Victor Vasarely, *Progresion 38 Blatt*
Victor Vasarely, *Vonal K SZ*

Part Two: An Op-Art Project: 2-Point Perspective/ Optical Illusion Project
The students will create a 2-point perspective drawing of their own; showing proficiency in using drawing and measuring tools, understanding pictorial space through the use of geometric shapes, variety of lines and tones in an optical illusionary manner. The composition will show a vanishing point and a grid formation. The composition will have parallel, perpendicular, intersecting, obtuse, acute, right angle, segmented, and a variety of other lines. Application of pen and the use of color will demonstrate *beginning skills* in developing 3-dimensional illusions through a choice of complimentary color schemes.

Materials
- a variety of measuring tools, rulers, t-square, pencil, triangles
- drawing paper, eraser and a variety of pencils; hb/2b
- it would be helpful to have a still-life of cubes, rectangles, and spheres (wooden math manipulative kit) if not available . . . a variety of size boxes, plus some beach balls, soccer and/or basketball
- markers and calligraphy marker pens

Concepts and Vocabulary to Research
Some vocabulary words here, are repeated for reinforcement of understanding.

Suggested 19th and 20th Century Artists
M.C. Escher, Piet Mondrian, Salvador Dali, Albert Albers, Victor Vasarely, Mike Field, and Dick Termes.

Terms
2-point perspective
multiple point perspective
complimentary color scheme
parameter
variable

symmetry	grid	value
Optical art/Op art	pictorial space	color tones

Ideas for the Center's Expansion

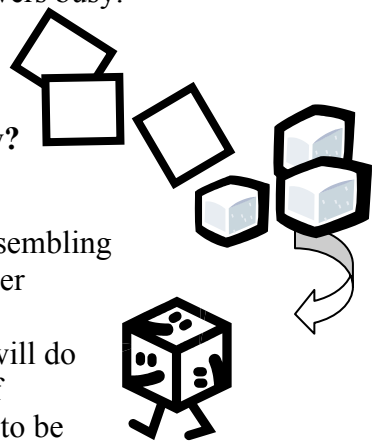
- Additional time to work on computer to create dimensional pictures; software suggestions: Geometry sketcher, Hyper-studio, Corel draw
- Lots of books with artists that use perspective as a parameter
- Free perspective drawing time, Let them draw!
- Give them a concept and/or vocabulary quiz to see if they are synthesizing the ideas

Suggested Web Sites for This Part of the Lesson

- <http://www.colorcube.com/illusions/optiart.htm>
- <http://www.vasarely.org/>
- <http://nothung.math.uh.edu/~patterns/>
- <http://www.ritsumei.ac.jp/~akitaoka/saishin-e.html>
- <http://www.art.com>
- <http://www.aaamath.com/geo318-polygons-numbers.html>

Excellent geometry worksheets that should be used for the lessons or for extra credit practice can be found at teaching sites such as <http://www.edHelper.com>. You'll be able to find many free worksheets that assist the lessons. This particular site has a worksheet that helps a student to classify lines, identifying perpendicular, parallel, and intersecting lines…. reinforcing the vocabulary introduced in the lesson, as well as many worksheets that are at higher levels to keep the overachievers busy.

Lesson 3:
2 Parts: Solving and choosing your own Parameters, Going from 2-D to 3-D . . . Having Fun with Geometry?



General Objectives and Time Allotment

The student will address learning deficiencies; acquire assembling skills and use of deductive reasoning, participate with other students to achieve a geometric construction, select and organize shapes, and learn new vocabulary. The student will do some research on their own… through books, film, use of computer software, and internet. This Lesson is designed to be a 1 month session.

Concepts and Vocabulary to Research

Some vocabulary words here are repeated for reinforcement of understanding.

Artists to Discover
Suggested 19th and 20th Century artists: Alexander Calder, M.C. Escher, Alan Linder, George Hart, Isamu Noguchi

Terms

Polygon	Polyhedra	Tessel	tiling	transition	reflection
rotation	translation	pattern	repetition	contrast	
positive & negative shapes		positive & negative concepts			
sculpture	parameter	variable	symmetry	variety of angles	
grid	pictorial space		tessellation		

Figures and Polygons for Discovery/ Shapes to Learn

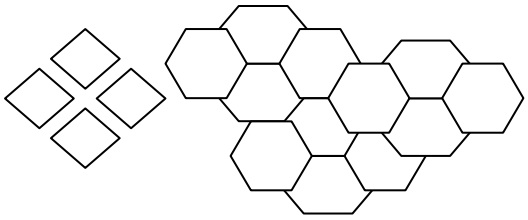
Polygon	Triangle	Equilateral triangle	
Isosceles triangle	Scalene triangle	Acute triangle	
Obtuse triangle	Right triangle	Quadrilateral	
Rectangle	Square	Parallelogram	
Rhombus	Trapezoid	Pentagon	
Hexagon	Heptagon	Octagon	
Nonagon	Decagon	Circle	Convex

A Good Polygon Resource for the Teacher (found in website reference):
- <http://www.mathleauge.com/help/geometry/polygons.htm>.

Project Objectives (for lesson 3, part I and II)
From the introduction of tiling planes and how mosaics work, show how 2d to 3d works in geometry. Students will distinguish, define, contrast, and interpret from polygons to polyhedral shapes.

Discovering tessellations, the artists, seeing how symmetry works in math and in art, the students will step up to produce, predict, segment, classify, and assemble a sculpture/composition, exploring Polyhedrons and their relationship to sculptures and architecture.

Provide opportunity to apply skills, concentrate on proficiencies, stressing intrinsic motivation. After the introductory work, information disseminated, and some experimentation… the student will show comprehension from composing, translate ideas, interpret and demonstrate by their own example of a handmade tessellation and a computer generated tessellation. They will have the opportunity to work with other students to create a geometric sculpture in a group.



Part One: Tessellations/ Polygon Project
The students will create a hand-made tessellation of their own; showing proficiency composition. plan. Show understanding pictorial space through the use of geometric shapes, and follow the parameters given. The teacher will assign whether the tessel will be a transition, rotational, reflection or a combination depending on the level of understanding and exposure the students have. After the first tessellation is created the student will create a computer-generated tessel with the software called "Tessel- mania."

Materials and Resources
- *Tessellations, How to create them* Video cassette, Crystal Productions Inc. 1999
- computer and software: *Tessel-mania*
- Internet
- variety of measuring tools, rulers, t-square, pencil, triangles
- drawing paper, eraser and a variety of pencils; h, and hb/2b,
- construction paper, glue and scissors

Centers Objectives
Encourage student to determine what to work on and accepts them as experts about their needs. Work completed at centers will earn extra credit, (an incentive for both students that need to have more comprehension and skills.)

Students will be able to work on different centers to have an opportunity to practice, discover, and become more proficient. Geometry centers will have additional projects and books for research. Students will also have the chance to go beyond their composition project and create another tessellation (their choice of parameter) for extra credit, time permitting.

For Re-enforcement

<u>Research from Reference Books</u>
- Have the student do some of the chapters in the math or art book that provide information for practice.
- Have students play the "Lesson in the box" (from centers-ideas, previously mentioned) creating geometric shapes with "Zome" a math construction manipulative.

<u>Research from Internet</u>
- Have students go to an interactive math program on the internet for fun.
- Have the student go on the internet to do some research on any of the subject matters introduced, depending of the student's need. Look up: Geometric shapes, George Hart, Calder's mobiles, math terminology, mathematicians or artist …. But have it be something *specific* to write about.

For Enrichment

- Have the student look up artist, Alan Linder, Isamu Noguchi or George Hart, Calder through the internet.
- Have the student create and compose another drawing with a sculpture or mobile similar to the artist style (provide materials for construction in this center.)

Suggested Web Sites for this Part of the Lesson

- <http://www.aaamath.com/geo318-polygons-numbers.html#section1>
- <http://www.aaamath.com/geo318-polygons-numbers.html>
- <http://www.georgehart.com/virtual-polyhedra/vp.html>
- <http://www.georgehart.com/index.html>
- <http://www.georgehart.com/zomebook/zomebook.html>
- <http://www.coolmath.com/interior.htm>
- <http://mathforum.org/dr.math/faq/faq.polygon.names.html>.
- <http://www.noguchi.org/geometric.html>
- <http://mathforum.org/mam/03/essay2.html>
- <http://www.allenlinder.com/>

Part Two – A Sculpture Project: Geometric Sculpting/ Experimentation Time

Given specific instructions to create a certain Polyhedron, the students will create a geometric form of their own; showing proficiency by using the "Zome" tool, understanding specific geometric shapes, be able to identify angles, turns/rotations (using fraction-knowledge) defend their "zome" creation by being able to speak about its name, shapes, angles, turns/rotation, and anything else to prove that they have completed the form correctly. This will be done by a short presentation to the class.

After a time exploring polyhedrons, the students will be assigned to a group that will create their own geometric form made by found objects. The group will have the ability to decide what materials they will use. It can be made from plastic utensils, noodles, spaghetti, string, dice, cards, cardboard, photographs, etc.…it's their choice! The material has to be accessible to the students, so that they can collect it and bring it in to construct it in the classroom. The sculpture/composition must have a whimsical name that makes the viewer/observer think. If they want to add color, it must be applied in a complimentary color scheme.

Materials

- a variety of measuring tools, rulers, t-square, pencil, triangles
- a collected bunch of found objects
- tape, glue, glue gun
- cardboard, construction paper
- scissors or utility knife
- band aids

ANNOTATED BIBLIOGRAPHY

Works Cited

Bouleau, Charles, and Jacques Villon. *The Painter's Secret Geometry: A Study of Composition in Art*. New York: Harcourt, Brace, and World, 1963.
It is a guide for serious art study, full of historical data from the past.

Cole, Alison, *Renaissance: Discover the Art of the Northern and Italian Renaissance, from the 14th to the 16th centuries*. New York: Dorling Kindersley Limited, 1994.
A good resource prepared in mind for the student. Large, clear, color pictures of essential information to understand the artist and theories of the times.

_____. *Perspective: A Visual Guide to the Theory and Techniques – From the Renaissance to Pop Art*. New York: Dorling Kindersley Limited 1992
Students will enjoy this illustrated book, large examples with interesting information about drawing perspective connecting art with mathematics.

Gregory, Gayle H. and Carolyn Chapman. *Differentiated Instructional Strategies: One Size Doesn't Fit All*. Thousand Oaks, CA: Corwin Press, Inc., 2002.
This guide allows teachers to choose from a variety of techniques, learn how to address the needs of individual students, and design strategies based on those needs.

Hine, Connie. *Developing Multiple Intelligences in Young Learners*. 1 May 2004. <http://www.i-watch.com.sg/i-watch/Html/Parenting/articles/MultipleInteligence.htm>.
An interesting online article that explains the benefits of providing and exposing children to meaningful learning.

Shalaway, Linda. *Learning to Teach . . . Not Just for Beginners: The Essential Guide for All Teachers*. Jefferson City, MO: Scholastic, Inc., 1998.
As the title makes clear, this book provides a wide range of advice and instruction for teachers of all levels of experience.

Teacher Resources

Bain, Ian. *Celtic Key Patterns*. Great Britain: Constable and Company Ltd, 1993.
From early Celtic patterns to Celtic patterns used in the Christian age.

Barrucand, Marianne and Achim Bednorz. *Moorish Architecture in Andalusia*. Hungary: Taschen Publishers, 1992.
A collection of significant architecture, showing Moorish influences of tile making, especially in early Spanish history.

Bates, Kenneth F. *Basic Design: Principles and Practice*. Cleveland: The World Publishing Company, 1960.
This compact book has a manner of looking at design in its most elemental form and compares it to geometric patterns.

Eicholz, Robert E., et al. *Addison-Wesley Mathematics: Grade 8*. CA: Addison-Wesley Publishing Company, 199l.
Standard eighth-grade math textbook. Newer versions/editions are available.

Eischolz Robert E., et al. *Addison-Wesley Mathematics: Grade 5*. CA: Addison-Wesley Publishing Company, 1989.
Standard fifth-grade math textbook.

Ghyka, Matila. *The Geometry of Art and Life*. New York: Dover Publications, Inc., 1977.
In finding geometry in natural objects we can then discover the mathematical side of seeing our world.

Giorgini, Frank. *Handmade Tiles: Designing, Making, Decorating*. New York: Sterling Publishing Company, 1994.
The techniques of planning, creating and placing of tile work.

Gregory, Gayle H. and Carolyn Chapman. *Differentiated Instructional Strategies: One Size Doesn't Fit All*. Thousand Oaks, CA: Corwin Press, Inc., 2002.

ISAMA 99: The International Society of *the Arts, Mathematics and Architecture*. Eds. Nathaniel A. Friedman and Javier Barrallo. Spain: University of the Basque Country, 1999.
For the more advanced interest in the three subject matters. Professionals and University professors got together to exchange thought. It provides a wide collection of papers, ranging in relationship to art, math and architecture. Description of this text would be that it is interdisciplinary, intuitive as well as exacting and scholarly in nature. The information provides very interesting observations and discussions about great art and architecture. For the more advanced in interest in these subject areas.

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Shows examples of how art and math have come together in our lives.

Schattschneider, Doris. *M.C. Escher: Visions of Symmetry*. New York: W.H. Freeman and Company, 1998.

A concise, clear, and illuminating text, the author unlocks Escher's images clearly, interestingly, for the layman.

Seymour, Dale and Jill Britton. *Introduction to Tessellations*. United States: Dale Seymour Publications, 1989.
How to tessellate, use polygons, see symmetry, develop transformations and other techniques shown in generating tessellations.

Stonesifer, Sue Ann. *Victorian Ornament and Pattern Designs to Color*. Owings Mills, MD: Stemmer House Publishers, Inc., 1982.
A small, but potent book, displaying many types of patterning created in the Victorian age.

Triado, Juan-Ramon. *The Key to Painting*. Minneapolis: Lerner Publications Company, 1988.
Small and concise, this book features Renaissance perspective tricks of the trade.

Supplemental Resources

Fantastic World of M.C. Escher. Directed by Michele Emmer. Videocassette. Crystal Video SVE Media, 1980. (50 minutes)
Tracing Escher's career and analyzing the math involved and seeing his genius, mathematicians and critics are asked to describe the world of this remarkable artist.

The Life and Works of M.C. Escher. Videocassette. Acorn Media Publishing Inc., 1999. (60 minutes)
Meticulous, complex, and precise are just some of the words to describe M. C. Escher. This film shows the fantastic imagery and extraordinary eye of the artist, trying to see how this artist ticked.

Tessellations: How to Create Them. Videocassette. Crystal Productions, Inc., 1999. (27 minutes)
Step-by-step instruction on how to create translations, reflections and rotational tessellations.

Student Resources

Cole, Alison. *Perspective: Discover the theories and techniques of perspective, from the Renaissance to Pop Art*. New York: Dorling Kindersley Limited, 1992
Students will enjoy this illustrated book, large examples with interesting information about drawing perspective connecting art with mathematics.

_____. *Renaissance: Discover the Art of the Northern and Italian Renaissance, from the 14th to the 16th centuries*. New York: Dorling Kindersley Limited, 1994.
A good resource prepared in mind for the student. Large, clear, colored pictures of essential information to understand the artist and theories of the times.

Math, Art, and Interactive Website Learning

Art – New Titles – Leonardo da Vinci – The Complete Paintings and Drawings – Facts. Taschen. 1 May 2004. <http://www.taschen.com/pages/en/catalogue/books/art/new/facts/01643.htm>.
All about artist and architects.

Conway, John. *Ask Dr. Math: FAQ: Naming Polygons and Polyhedra*. 1994-2004. The Math Forum. 1 May 2004. <http://mathforum.org/dr.math/faq/faq.polygon.names.html>.
Dr. Math is full of questions and answers for remembering polygon and polyhedra names! Great for student study of geometry vocabulary.

Field, Michael. *Homepage of Professor Mike Field*. 2004. Univ. of Houston. 1 May 2004. <http://www.math.uh.edu/~mike/>.
Symmetry, shapes, and much more from Mike Field's work

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Website for Isamu Noguchi's sculpture works.

Hart, George W. *Mathematical Awareness via Geometric Sculpture*. 1994-2004. The Math Forum. 1 May 2004. <http://mathforum.org/mam/03/essay2.html>.
A great website with examples of George Hart's sculptures

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An encyclopedia with many polyhedron forms that explains how they are formed and project ideas for students.

_____. *George W. Hart*. 1998-2003. George W. Hart. 1 May 2004. <http://www.georgehart.com/index.html>.
George Hart samples of his works. A contemporary artist/mathematician that creates modern geometric sculptures and writes books about geometry.

Hart, George W. and Henri Picciotto. *Zome Geometry: Hands-on Learning with Zome Models*. 1998-2003. George W. Hart. 1 May 2004. <http://www.georgehart.com/zomebook/zomebook.html>.

Perfect for student research! This website shows a lot of polyhedron samples made by the zome tool.

Interior Angles of Polygons. 1997-2004. Coolmath.com. 1 May 2004. <http://www.coolmath.com/interior.htm>.
A colorful and very friendly website for the student that needs elementary exposure to learning shapes.

Names of Polygons by Number of Sides and Angles. 2000. J. Banfill. 1 May 2004. <http://www.aaamath.com/geo318-polygons-numbers.html>.
A good interactive lessons and games for learning 2-D geometry

Spheres & Beings: Stone Carving by Allen Linder. 2003-2004. Allenlinder.com. 1 May 2004. <http://www.allenlinder.com/>.
A good website for examples of Allen Linder's sculpture

They will see the trees which were planted by their parents or grandparents. When they will reap the fruits or benefits of the tree, they will rejoice thinking about the great social work which their parents or grandparents did in their lives to ensure a brighter future for the ones who came after them. 4.3. 24 votes.Â Answer:unborn eyes refers to the future generation which is yet to be born, they would be happy to see that their ancestors had planted trees for the future generation's benefit. The future generation would get to enjoy the benefits of the tree that their ancestors had planted Explanation: 3.5.